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Neutrino Production in Extreme Conditions

massive neutrinos with magnetic moments in strong magnetic field and at high energy hadronic collision

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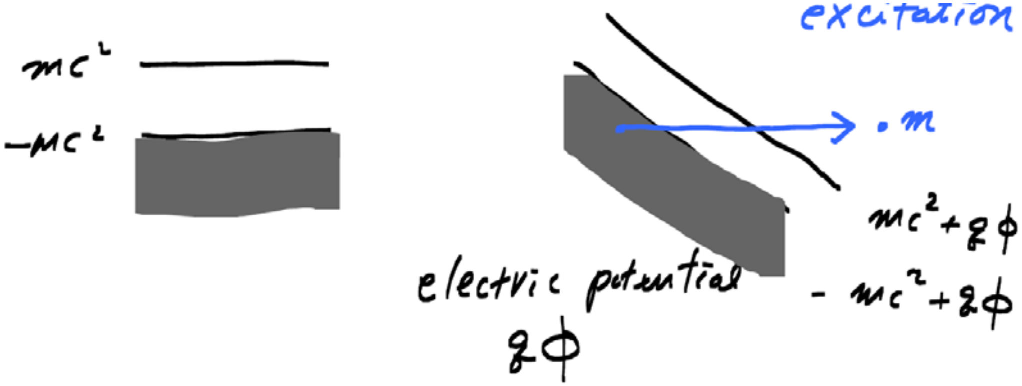
HKL & Y. Yoon , JHEP 0603,78(2006) ; 0703, 086(2007);
Mod. Phys. Lett. A 22, 2081(2007)
HKL, Y. Goh, W. Paeng, Y. Yoon, in preperation(2007)

I. Electromagnetic Vacuum Instability

1. QED e+ e- pair creation

Schwinger 1951



pair creation rate

$$w = w_0 \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-\pi n \left(\frac{E_c}{E}\right)}$$

$$w_0 = \frac{1}{4\pi^3} m^4 \left(\frac{E}{E_c}\right)^2$$

critical field strength

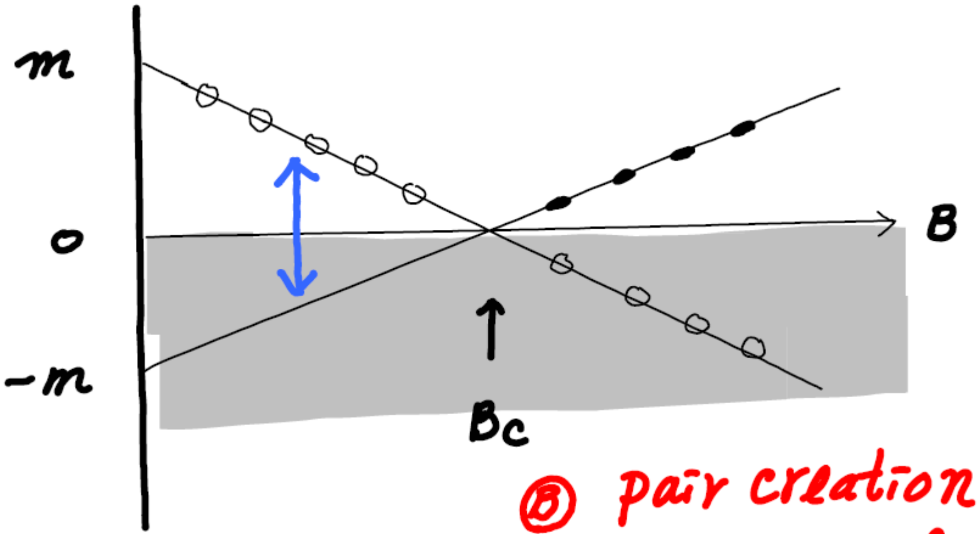
$$q E_c \cdot \lambda \sim mc^2$$

Electric effect

E^2 -B^2 > 0

2. Pair Creation of Neutral Fermion with Magnetic Moment

$$E = \pm(m - \mu B)\ , \quad P_t = P_l = 0$$



ΔE

Ⓑ pair creation in uniform B field

Ⓐ pair creation in non uniform B field

- $B_c = \dfrac{m}{\mu}$
- pair creation rate near B_c

$$w_c \sim m^4 = \frac{2\times 10^{35}\left(\frac{m}{eV}\right)^4}{m^3 \cdot s}$$

Pauli Interaction

- Neutral fermion with magnetic moment : μ

  Dirac-Pauli Lagrangian

$$\mathcal{L} = \bar{\psi}(\not{p} + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu} - m)\psi$$

$$E = \pm\sqrt{p_l^2 + (\sqrt{m^2 + p_t^2} \pm \mu B)^2}$$

Instability due to imaginary part of effective potential

Effective potential, V_{eff} :

$$i\int d^4x\, V_{eff}(A) = \int d^4x \langle x|\, tr \ln\left\{(\not p + \frac{\mu_a}{2}\sigma^{\mu\nu}F_{\mu\nu} - m)\frac{1}{\not p - m}\right\}|x\rangle$$

$$\Rightarrow V_{eff} = \frac{-i}{2}\int_0^\infty \frac{ds}{s} e^{-ism^2}\, tr\left(\langle x|e^{isp^2}|x\rangle - \langle x|e^{is(\not p + \frac{\mu_a}{2}\sigma^{\mu\nu}F_{\mu\nu})}|x\rangle\right)$$

- 'dual' property modulo γ-matrices:

Pauli	Minimal
$\vec{\nabla} \times \vec{B}$ $\longleftrightarrow$	$\vec{\nabla} \phi$
$\vec{\nabla} \cdot \vec{E}$ $\longleftrightarrow$	$\vec{\nabla} \times \vec{A}$

- Inhomogeneity of magnetic field plays analogous role of electric field.

$\Rightarrow$ Pair creation due to inhomogeneity.

$$B < B_c \left(= \frac{m}{\mu_B}\right)$$

$$V_{eff} = -\frac{\mu_a B}{4\pi^2}\int_0^\infty \frac{ds}{s}\left[i\int_0^1 d\xi(1-\xi)e^{i(\mu_a B)^2\xi^2 s} - \frac{i}{2} + \frac{1}{12}(\mu_a B)^2 s\right] e^{-im^2 s}$$

$$Im(V_{eff}) = -\frac{m^4\beta^2}{8\pi}\int_0^1 d\xi(1-\xi)\left[1-\beta^2\xi^2 - |1-\beta^2\xi^2|\right], \quad \beta = \frac{\mu_a B}{m}$$

$$= \frac{1}{48\pi}(|\mu_a B| - m)^3(|\mu_a B| + 3m)\,\theta(|\mu_a B| - m) \;!!$$

- $B < B_c$: no pair creation

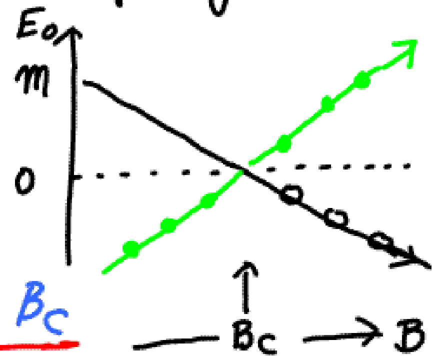
  $B \geq B_c$: pair creation in uniform magnetic field

e.g. $B = 2B_c$

$$Im(V_{eff}) = \frac{5}{48\pi}m^4$$

- Energy of ground state plunge into zero.

$$E_0 = m - \mu_a B$$



$\Rightarrow$ Pair creation beyond B_c ($B_c \equiv \frac{m}{\mu_a}$)

- minimal coupling (fermion)

$$E = \pm\sqrt{p_\ell^2 + m^2 + |eB|(2n+1 - \mathrm{sign}(e)\hat{s})}$$

ground state, $n=0$, $\hat{s}=-1$

$$E_0 = \sqrt{p_\ell^2 + m^2}$$

- Yang-Mills field (spin 1)

$$E = \pm\sqrt{p_\ell^2 + |gB|(2n+1 - 2\hat{s})}$$

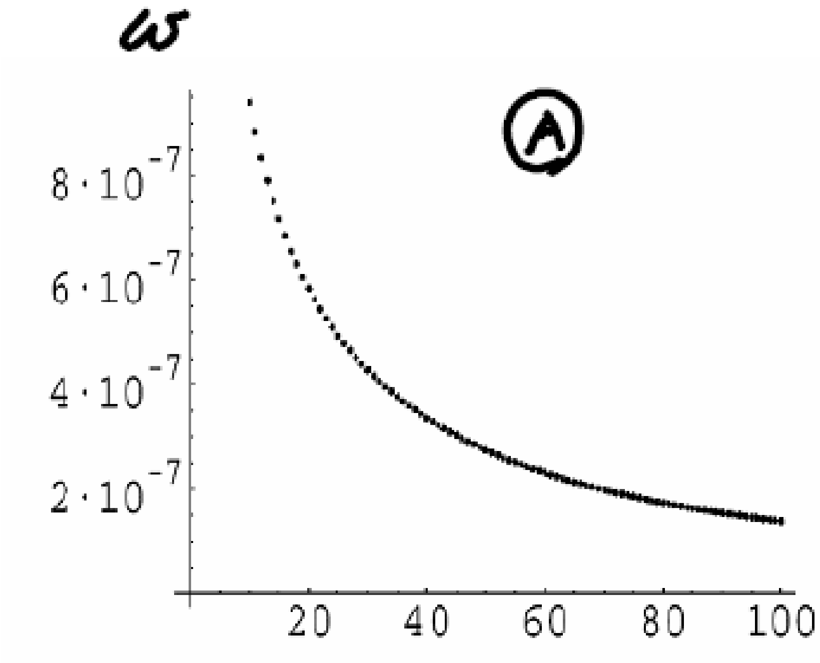
$E_0 \longrightarrow 0$ for ground state

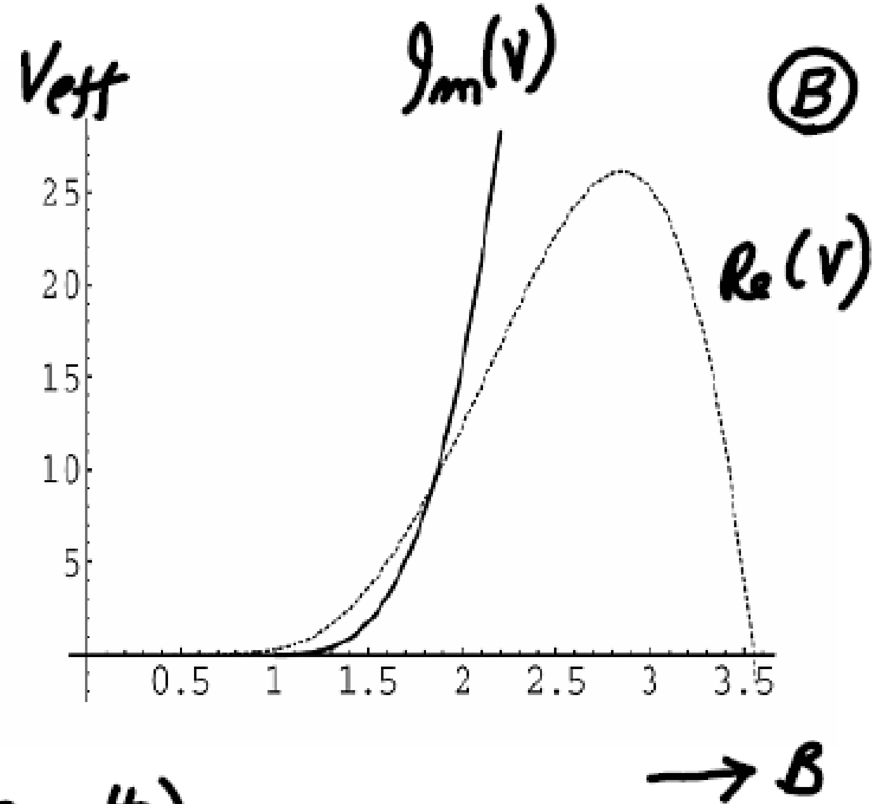
$\Rightarrow$ Unstable Yang-Mills mode

Savvidy 97, Nielsen-Olesen 98

...

Pair Creation as Magnetic Effect
$B^2-E^2>0$





$\{\omega, V_{eff}\} / (m^4/48\pi^4)$

HKL & Y. Yoon JHEP 03,78(2006)

HKL & Y. Yoon JHEP 03, 086(2007)

3. Pair creation of Majorana neutrino with transition magnetic moment

Neutrino:

1. electrically neutral(beta decay)
2. massive(neutrino oscillation)
3. flavor mixing (neutrino oscillation)
4. Dirac or Majorana ?
5. magnetic moment ?
6. physics beyond standard model

Dirac-Pauli Lagrangian for Majorana neutrinos with transition magnetic moment

- Mass eigenstates,

$$M = m_1 + m_2 \quad \Delta m = m_1 - m_2$$

- Majorana

$$\psi_c = \psi, \quad \psi_c = C\bar{\psi}^T$$

- Transtion magnetic moment $\mu_A \sigma_2$

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ - \frac{1}{2}M - \frac{1}{2}\Delta m \sigma_3 + \frac{\mu_A}{2}\gamma^{\mu\nu}F_{\mu\nu}\sigma_2)\psi$$

[I] For $b^2 \le 1$,

$$\Im V_{\rm eff} = 0$$

$$b \equiv \left|\frac{2\mu_A B}{M}\right|$$

$$\delta = \frac{\Delta m}{M}$$

[II] For $1 \le b^2 \le 1 + (\sqrt{2}-1)^2\delta^2$,

$$\Im V_{\rm eff} = \frac{M^4\delta^2}{64\pi}(b^2-1).$$

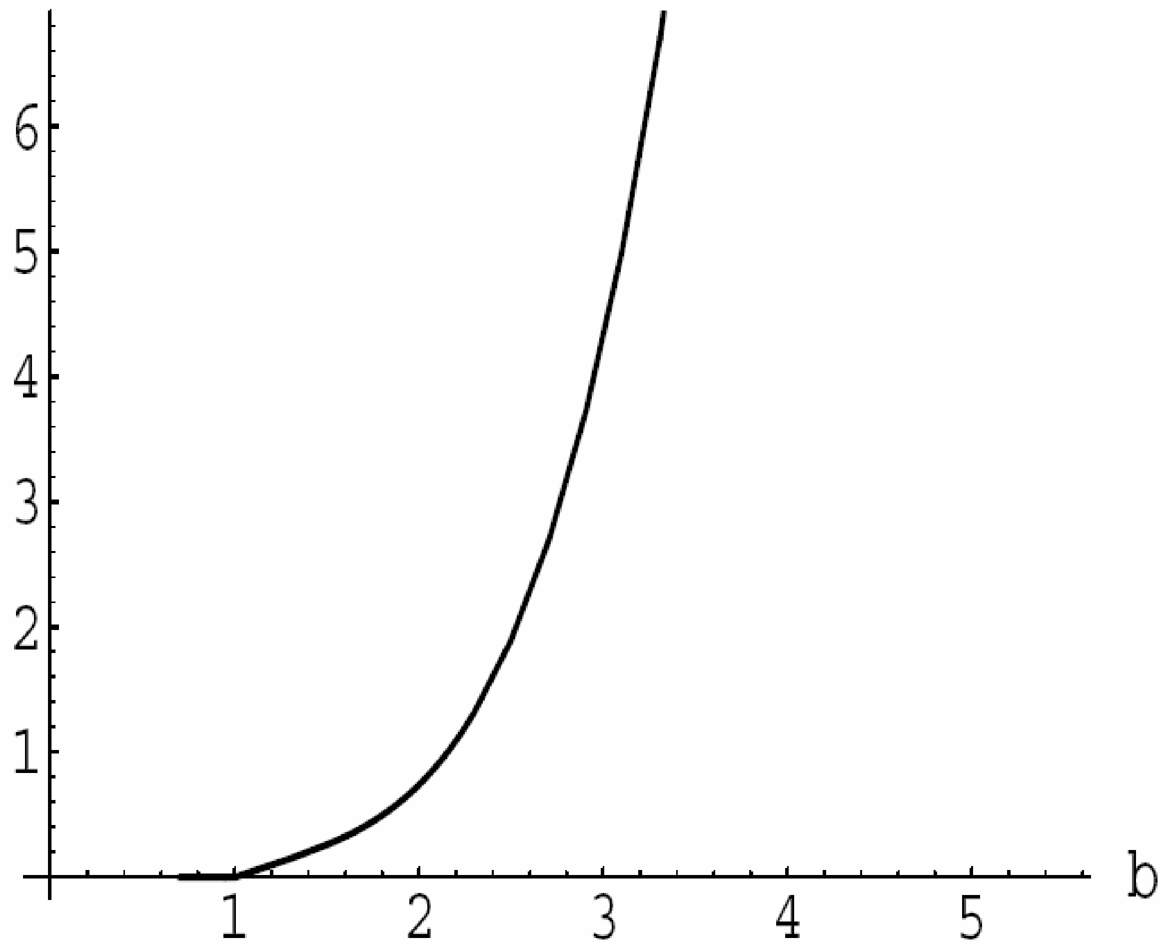
[III] For $1 + (\sqrt{2}-1)^2\delta^2 \le b^2 \le 1 + (\sqrt{2}+1)^2\delta^2$,

$$\Im V_{\rm eff} = \frac{M^4}{1536\pi}[b^4 - 6b^2(1-\delta^2) - 3(1+6\delta^2+\delta^4)$$
$$+8b\{(1+2\delta^2-\frac{\delta^2}{b^2})^{3/2} - (\delta^2-\frac{\delta^2}{b^2})^{3/2}\}].$$

[IV] For $1 + (\sqrt{2}+1)^2\delta^2 \le b^2$,

$$\Im V_{\rm eff} = \frac{M^4}{768\pi}\{b^4 - 6b^2(1+\delta^2) - 3(1+\delta^2)^2 + 8b(1+2\delta^2-\frac{\delta^2}{b^2})^{3/2}\}$$

64π Im(V)/M^{4} for δ=1/2



HKL & Y. Yoon, in preparation 2007

Summary(I)

- Theoretical and experimental bounds on neutrino magnetic moment $\mu_\nu < 10^{-9} \sim 10^{-12} \mu_B$
- Critical magnetic field $B_c \sim 10^{15} G \left(\frac{m_\nu}{10^{-2} eV}\right)\left(\frac{10^{-9}\mu_B}{\mu_\nu}\right)$
- Slow process $\sim (m_\nu / m_e)^4$
- Continuous source of neutrinos

$$10^{39} s^{-1} \left(\frac{m_\nu}{10^{-2} eV}\right)^4 \left(\frac{B}{B_c}\right)^4 \left(\frac{R}{10 Km}\right)^3$$

- Production in mass eigenstate
- Model for neutrino magnetic moment
- Effective lagrangian for strong magnetic field; $\mu_\nu > 10^{-19} \mu_B$

II. Neutrino Production in High Energy Hadron Collision

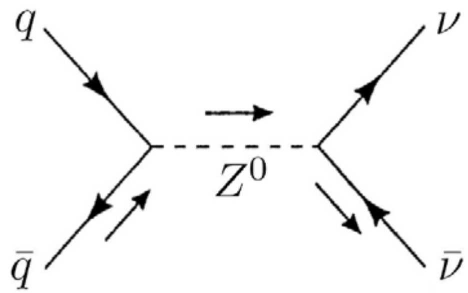
- High energy hadron collision

$E_{CM}^{LHC} \sim 10\ TeV$

$E_{CM}^{UHECR} \gtrsim 10^2\ TeV$

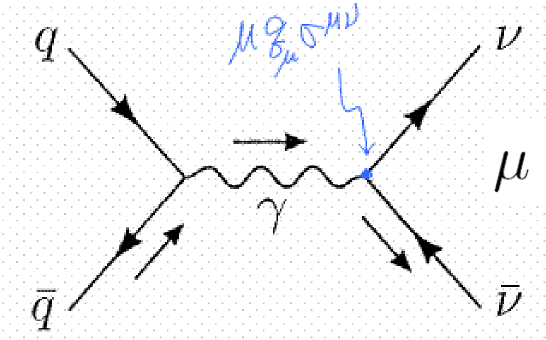
- Neutrino pair creation in standard model

$$\sigma_{SM} = 6.1 \times 10^{-43} m^2 \left(\frac{TeV}{E} \right)^2$$

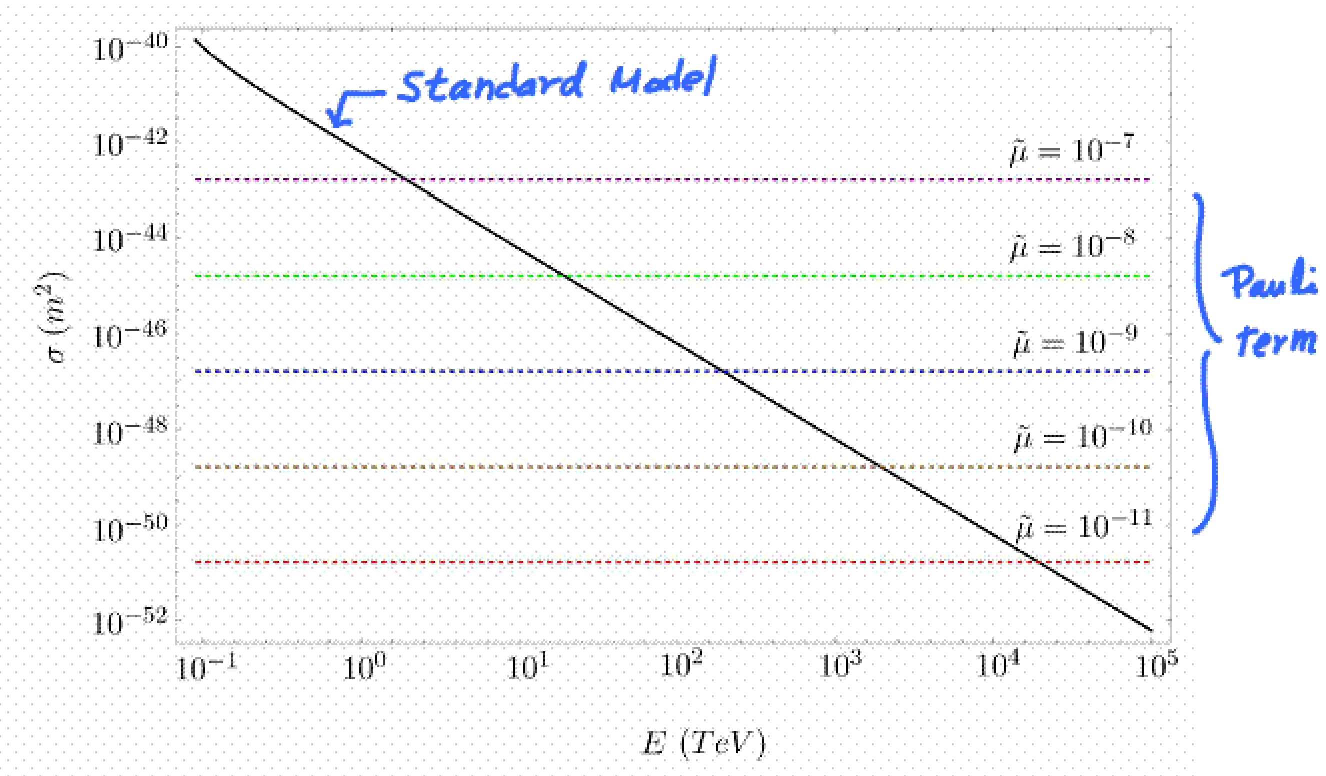


- Neutrino pair production through Pauli interaction

$$\sigma_P = \frac{2}{3}\alpha\mu^2$$
$$= 1.64 \times 10^{-29} m^2 (\tilde{\mu})^2$$



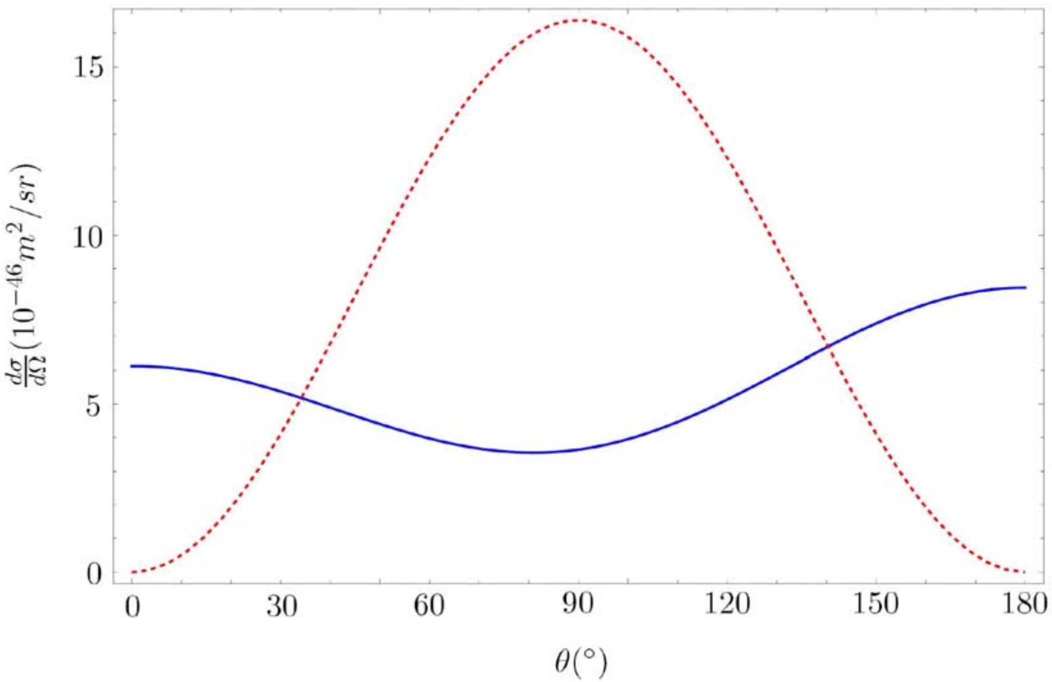
$\mu \equiv \tilde{\mu}\mu_B$



- Angular distribution

$$\left(\frac{d\sigma}{d\Omega}\right)_P = \frac{\alpha\mu^2}{4\pi}\sin^2\theta$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{SM} = 2.89\times 10^{-43}\times\{0.13(1+cos^2\theta) - 0.04cos\,\theta\}\left(\frac{m^2}{sr}\right)\left(\frac{TeV}{E}\right)^2$$

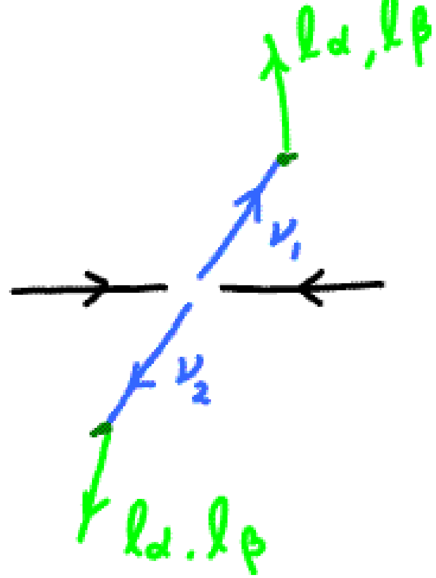


$E_{CM} = 10\ TeV$

$\hat{\mu} = 10^{-8}$

- Production in mass eigenstate

$$\nu_1 = \cos\theta\, \nu_\alpha + \sin\theta\, \nu_\beta$$
$$\nu_2 = -\sin\theta\, \nu_\alpha + \cos\theta\, \nu_\beta$$



- Back-to-back correlation of charged lepton production.

$$R_- \equiv \frac{N_1(l_\alpha) - N_1(l_\beta)}{N_2(l_\alpha) - N_2(l_\beta)} = -1$$

$$R_+ \equiv \frac{N_1(l_\alpha) - N_1(l_\beta)}{N_2(l_\alpha) + N_2(l_\beta)} = \cos^2\theta - \sin^2\theta$$

Possible Observabilities with Neutrino Magnetic Moment

-Neutrino production in
Ultra-Strong Magnetic Field, B > 10^{15}G
(Soft Gamma Repeaters,
Anomalous X-Ray Pulsars, Neutron stars,
Black holes, …)

-Enhancement of neutrino production in high energy hadronic collision (LHC, UHECR)
(Angular distribution, Back-to-back correlation, …)

- Useful numbers

	neutron	neutrino	electron	...
m	940 MeV	$\sim 10^{-2}$ eV	0.5 MeV	
μ_a	$-1.9\,\mu_N$	$< 10^{-11}\mu_B$	$\frac{\alpha}{2\pi}\mu_B$	
$w \sim m^4$	$10^{66}/m^3 \cdot s$	$\sim 10^{27}/m^3 \cdot s$	$10^{54}/m^3 \cdot s$	
$B_c = \frac{m}{\mu_a}$	10^{23} G	$> 10^{17}$ G	10^{17} G	

- Indication of maximum field strength of magnetic field

$\Rightarrow$ full higher order loop calculation is needed !!

- In QED, Pauli-Interaction description of anomalous magnetic moment of charged fermion is only valid for $B \ll B_c$.

(B. Jancovici, PR 187, 2275 (69), ...)