Oscillating Quintessence

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Quintessence (based on FLRW)

Quintessence: dynamical scalar field ϕ

Action : $$S = \int dx^4 \sqrt{|g|} \left[\frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right]$$

Field equation: $$\frac{\partial^2 \phi}{\partial t^2} + 3H \frac{\partial \phi}{\partial t} - \frac{1}{a^2} \nabla^2 \phi + V'(\phi) = 0$$

energy density and pressure :

$$\rho_\phi = \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2a^2} (\nabla \phi)^2 + V(\phi)$$

$$p_\phi = \frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{6a^2} (\nabla \phi)^2 - V(\phi)$$

Slow evolution and weak spatial dependence
➔ $V(\phi)$ dominates ➔ $w_\phi \sim -1$ ➔ Acceleration

? **How to achieve it** (naturally) **?**

Quintessence (FLRW)

Quintessence: dynamical scalar field ϕ

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Assume $\phi = \phi(t) \;\Rightarrow\; \rho_\phi = K + V\,,\; p_\phi = K - V\,,$ where $K \equiv \frac{1}{2}\dot{\phi}^2$

$$\Rightarrow w_\phi = p_\phi \big/ \rho_\phi = (K - V) \big/ (K + V)$$

Quintessence: oscillation —?→ repulsive gravity

straightforward naïve thinking:

oscillation	$\rightarrow$	excitation particle	$\rightarrow$	$p \geq 0$ attractive gravity	—X→	cosmic acceleration

(example) $V(\phi) = \frac{1}{2} m^2 \phi^2$:

an oscillating mode : $\phi \sim \phi_0 \cos(kx \pm \omega_k t + \theta)$, $\omega_k^2 = \frac{k^2}{a^2} + m^2$

$$\langle \rho_\phi \rangle_t \cong \frac{1}{2}\left(\frac{k^2}{a^2} + m^2\right)\phi_0^2 \quad , \quad \langle p_\phi \rangle_t \cong \frac{1}{6}\frac{k^2}{a^2}\phi_0^2 > 0$$

$$w_\phi = \langle p_\phi \rangle_t \big/ \langle \rho_\phi \rangle_t \cong \begin{cases} \frac{1}{3} \ , & \text{for } \frac{k^2}{a^2} >> m^2 \ \text{(large-}k\text{ mode)} \\ 0 \ , & \text{for } \frac{k^2}{a^2} << m^2 \ \text{(small-}k\text{ mode)} \end{cases}$$

~ radiation

~ NR matter

Quintessence: a slowly evolving mode

For $V(\phi) = \frac{1}{2} m^2 \phi^2$:

solution : $\phi \sim \phi_0 \cos(kx \pm \omega_k t + \theta)$, $\omega_k^2 = \frac{k^2}{a^2} + m^2$

- weak spatial dependence : $k \sim 0$

  slow evolution : $\omega_{k\sim 0} \leq H_0$

  → $m \leq H_0 \sim 10^{-48}\,\text{GeV}$ (extremely small !!)

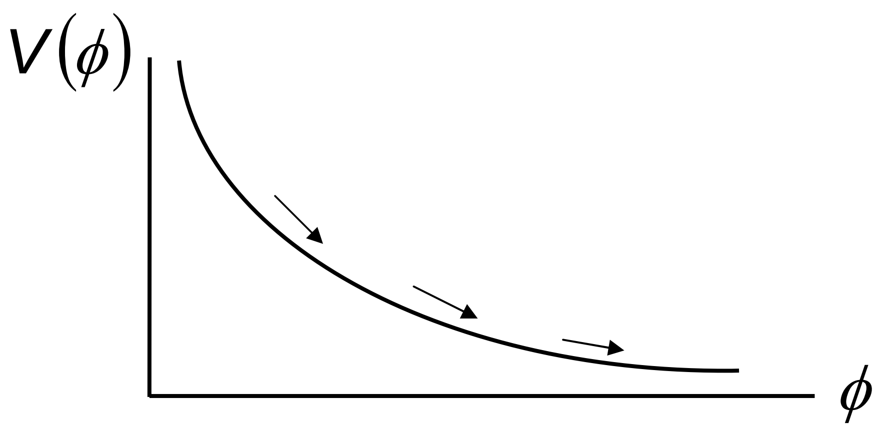
- $V(\phi) = \frac{1}{2} m^2 \phi^2 \sim \rho_\phi \sim \rho_c \equiv \frac{3H_0^2}{8\pi G}$ [$V(\phi)$ dominates.]

→ $\phi \geq G^{-1/2} \sim$ Planck energy $\sim 10^{19}\,\text{GeV}$ (extremely large !!)

Quintessence: oscillation —?→ repulsive gravity

It seems that oscillators for repulsive gravity (dark energy).

Accordingly, instead of oscillation-like modes,
most quintessence models involve **"slowly" moving modes**
such as **running away behavior**:



(example) **tracker quintessence**

- Power-law : $V(\phi) = \dfrac{M^{4+n}}{\phi^{n}}$
- Exponential : $V(\phi) = V_0 e^{-\phi/M}$

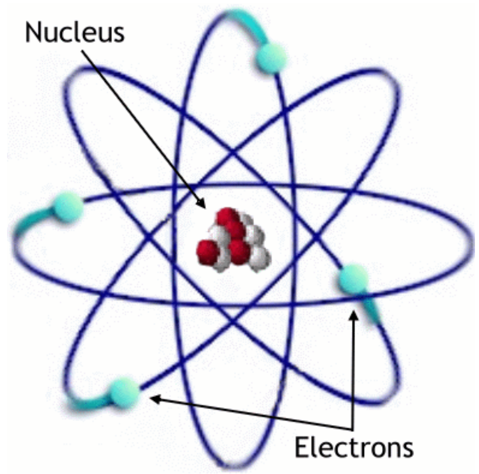
$\phi \to \infty$ $(V \to 0)$ **asymptotically.**

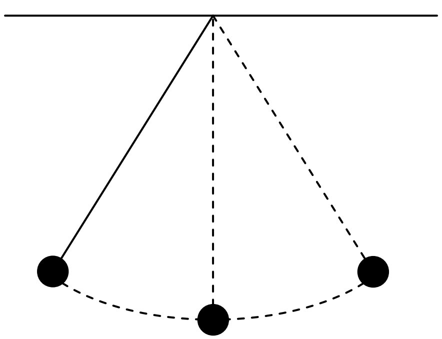
Quintessence: oscillation —?→ repulsive gravity

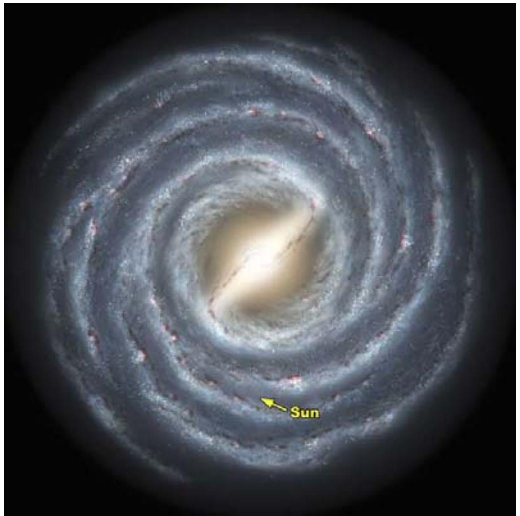
? **oscillation —X→ repulsive gravity (dark energy) !?**

If so, pity…… (oscillation-like behavior so frequent/familiar/friendly)

(for example)







Quintessence: oscillation —?→ repulsive gravity

In this talk, show

Oscillating Quintessence

↓

negative pressure
repulsive gravity
dark energy

Oscillating Quintessence

Action: $S = \int dx^4 \sqrt{|g|}\,[K - V]$

Field equation: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$\begin{bmatrix} \text{ignoring spatial dependence} \\ \text{ignoring spatial curvature } (k/a^2) \end{bmatrix}$

power - law potential : $V(\phi) = M^{4-n}|\phi|^n$, $n > 0$, $M > 0$

$\left(\text{regularization for } 0 < n \le 1: \;\; V(\phi) = \frac{M^{4-n}|\phi|^{1+n}}{|\phi| + \varepsilon} \right)$

For oscillation, consider

time-averaged quantities (averaged over a period T): $\langle \rho_\phi \rangle, \langle p_\phi \rangle, \langle K \rangle, \langle V \rangle \ldots$

$$\Rightarrow \; \langle K \rangle = \frac{n}{2} \langle V \rangle \quad \Rightarrow \; w_{OQ} = \frac{n-2}{n+2}$$

(simply determined by *n*)

n	4	2	1	0.1	0^+
w_{OQ}	$1/3$	0	$-1/3$	-0.9	-1^+

$$\Rightarrow \; 0 < n < 1 \Leftrightarrow -1 < w_{OQ} < -1/3 \;\; \text{(repulsive gravity)}$$

Oscillating Quintessence

(other quantities)

$$\rho_{OQ} \equiv \langle \rho_\phi \rangle = \rho_c \Omega_{DE} (1+z)^{3(1+w_{OQ})} = V_{\max} = M^{4-n} |\phi|^n_{\max}$$

$$\Omega_{DE} = \rho_{OQ}(0) \big/ \rho_c = M^{4-n} |\phi|^n_{\max}(0)$$

$$\text{period } T = \frac{\sqrt{8\pi}}{n} \frac{\Gamma(1/n)}{\Gamma(1/n + 1/2)} M^{1-4/n} \rho_{OQ}^{1/n-1/2}$$

$$HT = \frac{8\pi}{\sqrt{3}n} \frac{\Gamma(1/n)}{\Gamma(1/n + 1/2)} M_{pl}^{-1} M^{1-4/n} \rho^{1/2} \rho_{OQ}^{1/n-1/2} = \frac{8\pi}{\sqrt{3}n} \frac{\Gamma(1/n)}{\Gamma(1/n + 1/2)} \sqrt{\frac{\rho}{\rho_{OQ}}} \frac{|\phi|_{\max}}{M_{pl}}$$

$$H_0 T_0 = \frac{\sqrt{3}}{n} \frac{\Gamma(1/n)}{\Gamma(1/n + 1/2)} \left(\frac{3}{8\pi}\right)^{4/n-1} \Omega_{OQ}^{1/n-1/2} \left(\frac{M}{M_{pl}}\right)^{1-2/n} \left(\frac{M}{H_0}\right)^{-2/n}$$

$$= \frac{8\pi}{\sqrt{3}n} \frac{\Gamma(1/n)}{\Gamma(1/n + 1/2)} \Omega_{OQ}^{-1/2} \frac{|\phi|_{\max}(0)}{M_{pl}}$$

Oscillating Quintessence

- Constraints from observations:

$$w_{OQ} < -0.9\,(1\sigma) \quad (\text{for a const. } w_{DE})$$
$$\Omega_{DE} \cong 0.75$$

$$\Rightarrow \begin{cases} 0 < n < 0.1 \\ V_{max}(0) = M^{4-n}|\phi|^n_{max} \cong 0.75\rho_c \cong 3\times 10^{-11}\text{eV}^4 \end{cases}$$

$$\left(\underline{\text{rem}} \quad w_{OQ} = \frac{n-2}{n+2}\right)$$

- Condition for oscillation: $HT << 1$ from now back to some early time

$$H_0T_0 << 1 \Rightarrow \begin{cases} M >> M_{min} \equiv \left[\frac{8\pi}{\sqrt{3n}}\frac{\Gamma(1/n)}{\Gamma(1/n+1/2)}\Omega_{OQ}^{1/n-1/2}\rho^{1/n}M_{pl}^{-1}\right]^{n/(4-n)} \\ |\phi|_{max}(0) << \text{Ampl.}(0)_{max} \equiv \frac{8\pi}{\sqrt{3n}}\frac{\Gamma(1/n)}{\Gamma(1/n+1/2)}\sqrt{\Omega_{OQ}}M_{pl} \sim M_{pl} \end{cases}$$

$$HT << 1 \quad \Rightarrow \quad z << z_{max} \equiv F_n^{-1}(1/H_0T_0) \quad \text{for given } H_0T_0$$

z_{max} ~ oscillation starting time

$$F_n(z) \equiv (1+z)^{3(2-n)/(2+n)}\sqrt{\rho/\rho_c}$$

Oscillating Quintessence

(example) $n = 0.1\ ,\quad w_{OQ} = -0.9$

$$M >> M_{\min} = 4.3 \times 10^{-4}\,\mathrm{eV}\ ,\qquad \frac{|\phi|_{\max}(0)}{M_{pl}} << \frac{\mathrm{Ampl}(0)_{\max}}{M_{pl}} = 40$$

M	$(\rho_c^{1/4})$ $10^{-2.6}\,\mathrm{eV}$	eV	GeV	TeV	(GUT) $10^{15}\,\mathrm{GeV}$	(M_{pl}) $10^{19}\,\mathrm{GeV}$
$Log\left[\frac{|\phi|_{\max}(0)}{M_{pl}}\right]$	-32	-133	-484	-601	-1069	-1225
$Log[H_0 T_0]$	-30	-131	-482	-599	-1067	-1223
$Log[z_{\max}]$	6.9	28	103	128	227	260

Summary

- Feasibility of **OK**

  oscillation $\longrightarrow$ repulsive gravity $\longrightarrow$ cosmic acceleration

  (dark energy, enough negative pressure)

  $$\left(\begin{array}{l} \text{opposite to the naive thinking:} \\ \text{oscillation} \rightarrow \begin{array}{c}\text{excitation}\\ \text{particle}\end{array} \rightarrow \begin{array}{c} p \geq 0 \\ \text{attractive} \\ \text{gravity} \end{array} \not\longrightarrow \begin{array}{c}\text{cosmic}\\ \text{acceleration}\end{array} \end{array} \right)$$

- w_{OQ} is simply determined by the power n:

  $$w_{OQ} = \frac{n-2}{n+2}$$

n	4	2	1	0.1	0^+
w_{OQ}	$1/3$	0	$-1/3$	-0.9	-1^+

- Requirements from the oscillation condition and the observational constraints are presented.

Extend the scope of DE model construction.

For models involving scalar field(s) such as quintessence (even other fields)