

Cosmological Constant as a Manifestation of the Hierarchy

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PC and Je-An Gu, astro-ph/0409238 (2004)

PC, astro-ph/06112378 (2006)

PC and Je-An Gu, in preparation (2007)

Cosmological Constant Problem

1. The smallness problem
Why is $CC \ll SM$?
2. The concordance problem
Why is $\Omega_\Lambda \sim \Omega_m \sim \Omega_{\text{total}}$?

Coined by Michael Turner as the

“Nancy Keragan problem: Why me, why now?”

Another Hierarchy in Physics

- Gravity is much weaker, or Planck scale (10^{19} GeV), much larger, than that of SM gauge interactions:

$$\frac{M_{PI}}{M_{SM}} \sim 10^{16} .$$

Two well-known solutions:

ADD : large (but flat) extra dimension

RS : warped geometry

A Numerical Coincidence

- A remarkable numerical coincidence,

$$\frac{M_{CC}}{M_{SM}} \simeq \frac{M_{SM}}{M_{Pl}} \equiv \alpha_G$$

- Perhaps not accidental but implies a deeper connection:

$$M_{CC} \simeq \frac{M_{SM}}{M_{Pl}} M_{SM} = \left(\frac{M_{SM}}{M_{Pl}} \right)^2 M_{Pl} = \alpha_G^2 M_{Pl}.$$

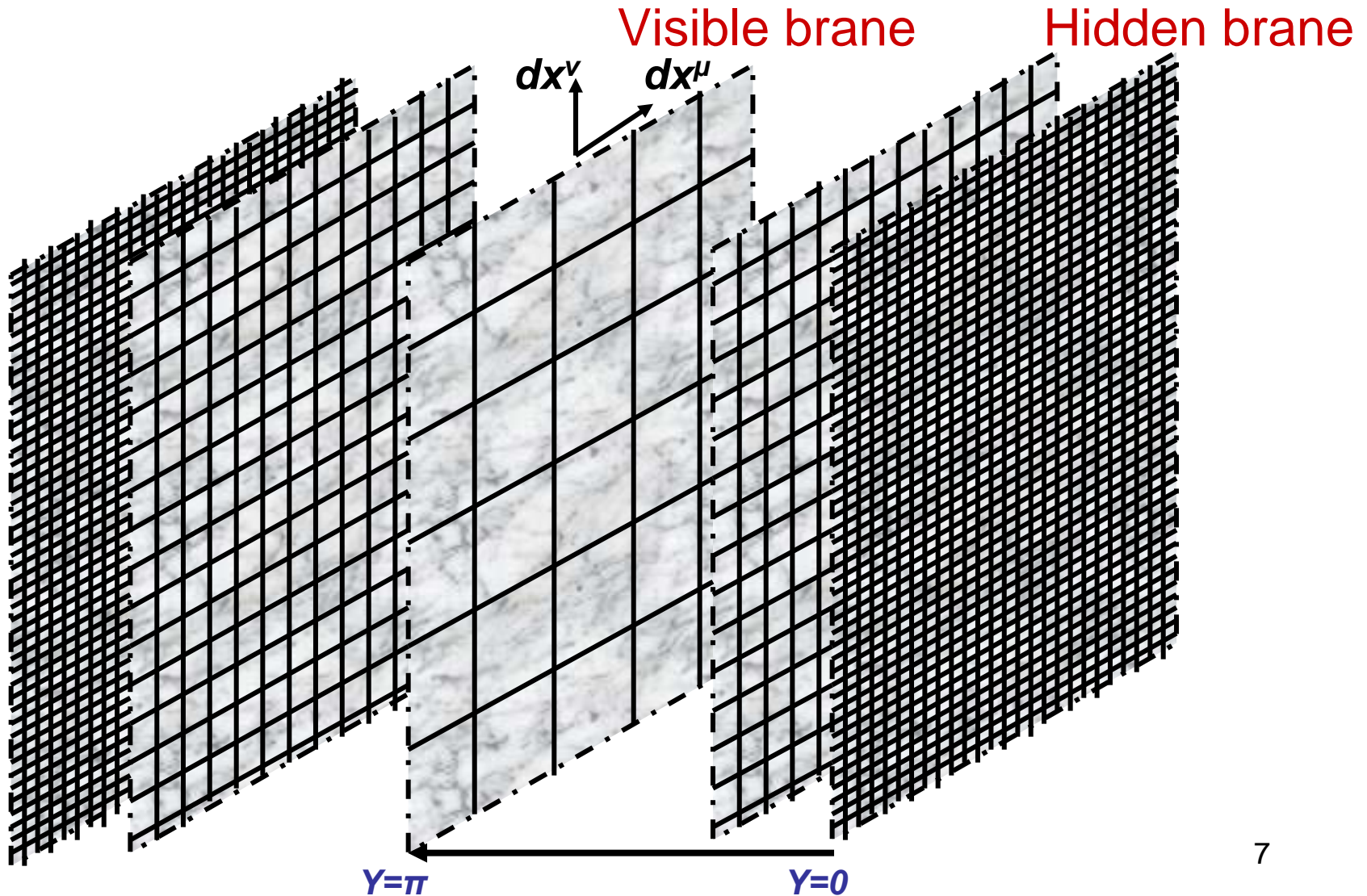
- **Caution:** Unlike the 1st hierarchy that links 4 fundamental interaction strengths, DE must be a secondary, derived quantity.

Analogy in Atomic Physics

- Bohr atom
- Fundamental energy scale in Schrodinger equation: m_e
- Ground state energy suppressed by 2 powers of fine structure constant
- Dark energy
- Fundamental energy scale in quantum gravity: M_{Pl}
- Dark energy suppressed by 2 powers of “gravity fine structure constant”

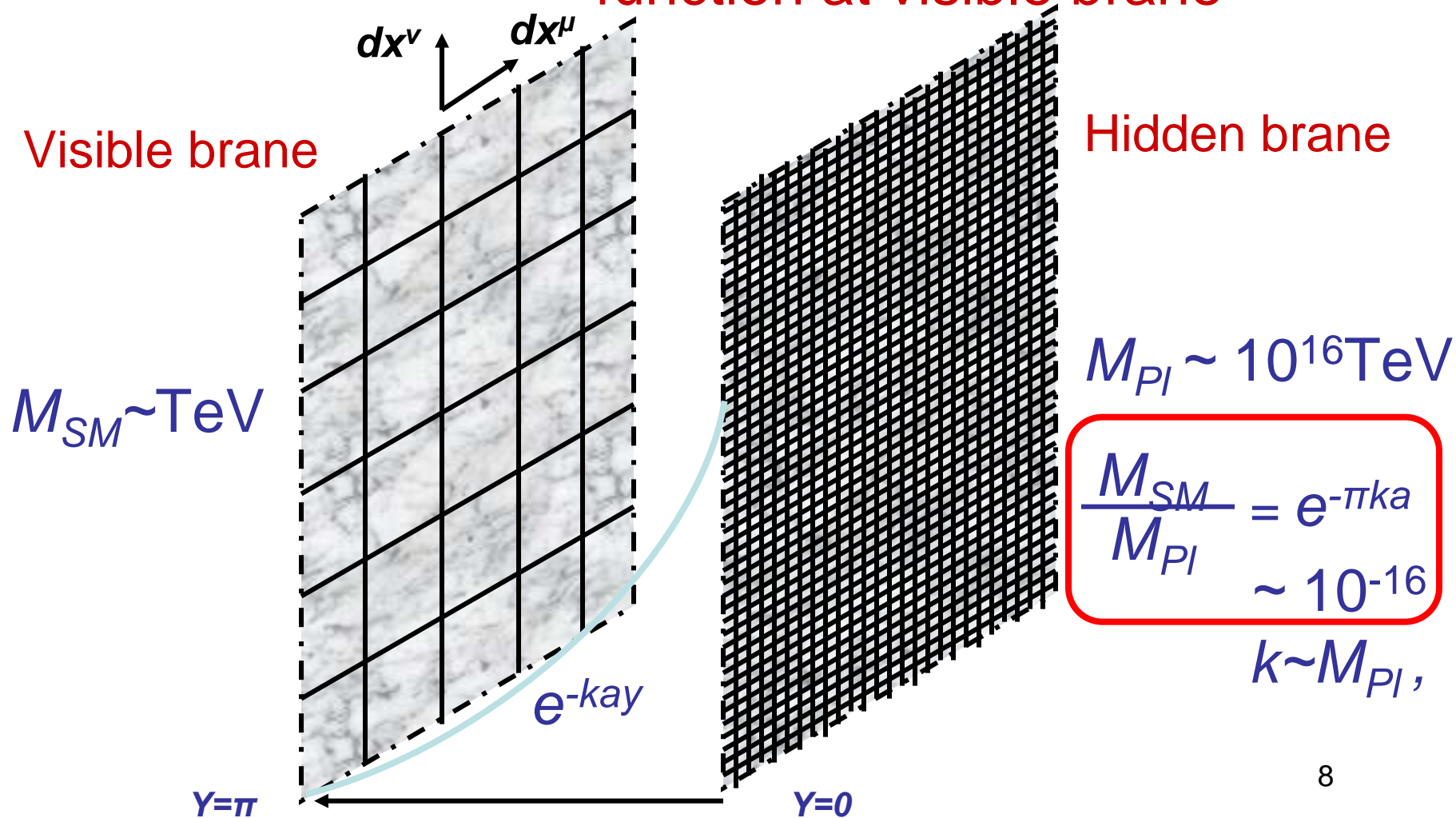
Randall-Sundrum Warped Geometry

$$ds^2 = e^{-2\sigma} \eta_{\mu\nu} dx^\mu dx^\nu + a^2 dy^2$$

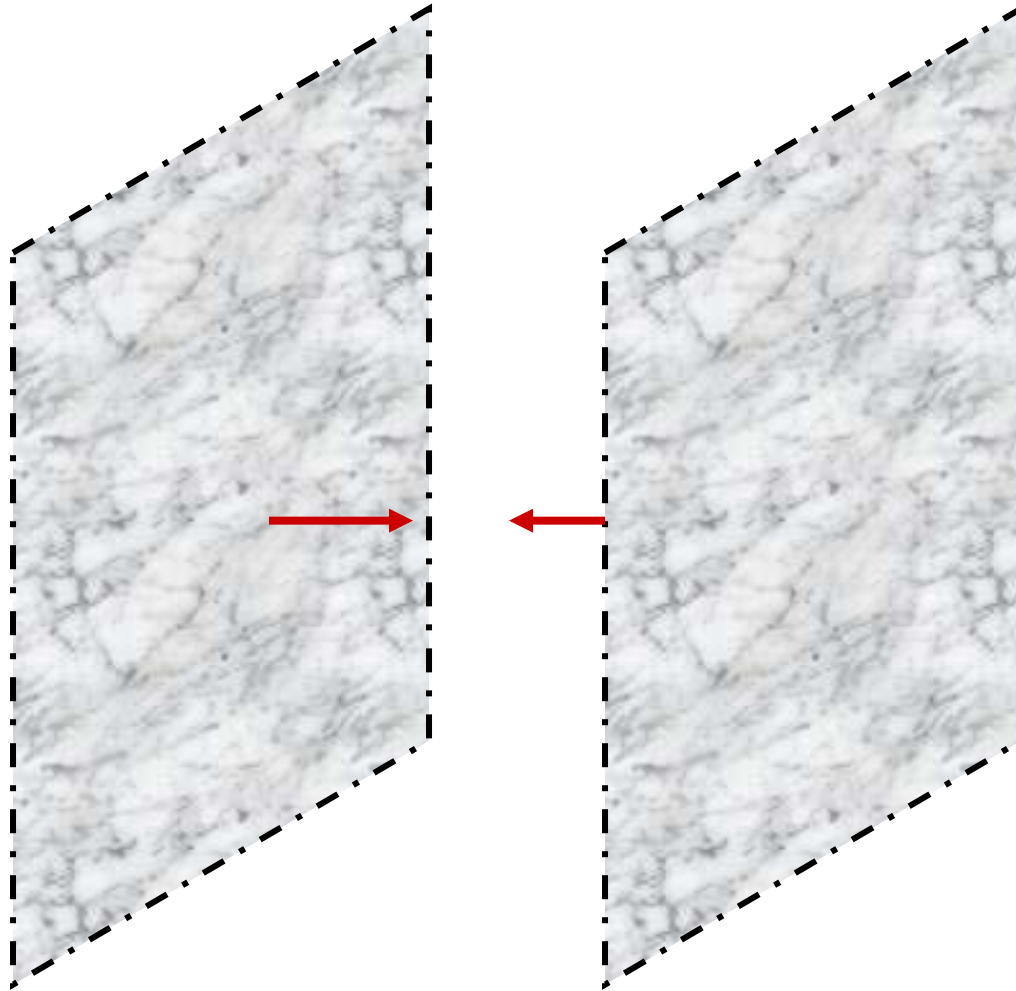


Randall-Sundrum Warped Geometry

Weakness of gravity = smallness of graviton wave function at visible brane

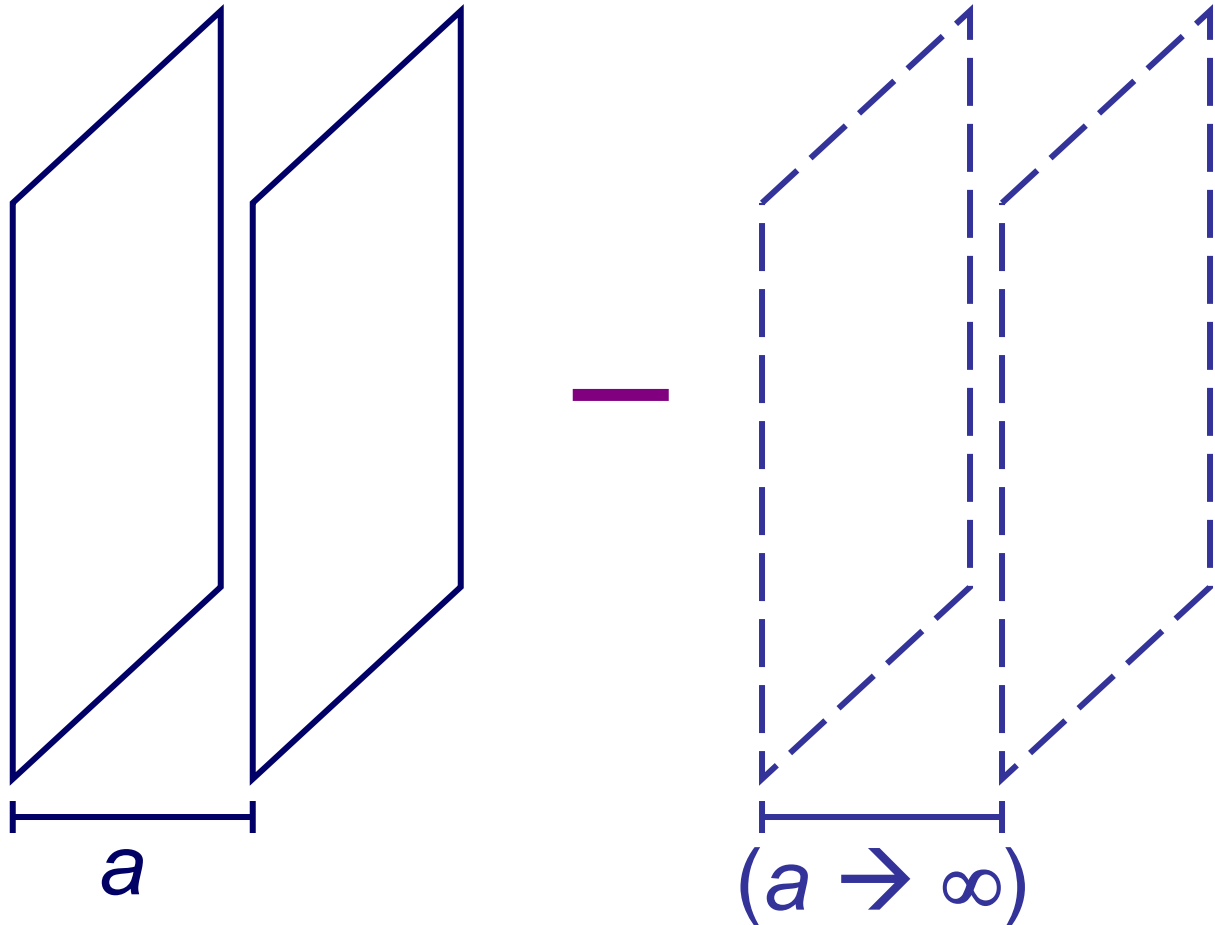


Casimir Effect



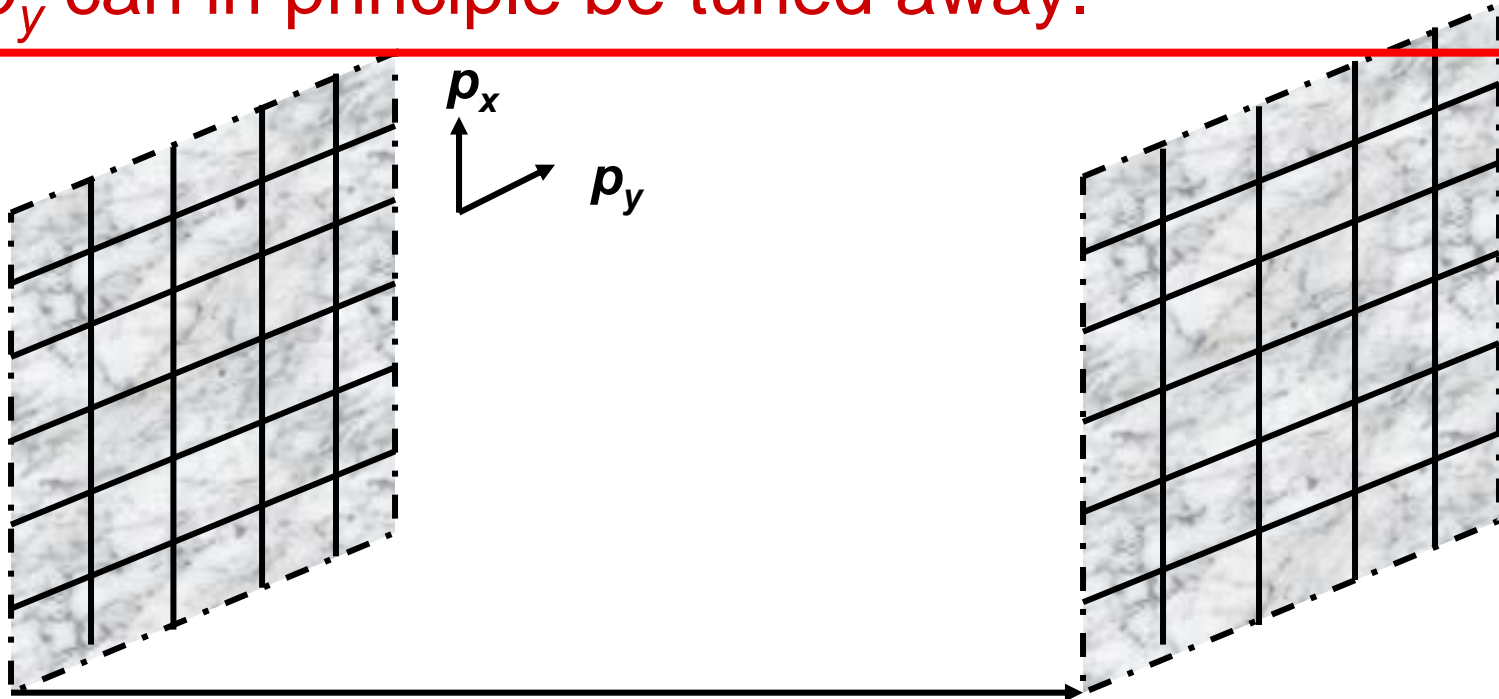
Casimir Energy

$$\rho_{\text{casimir}} = \rho_{\text{vac}} \left(\left| \quad \right| \right) - \rho_{\text{vac}}(a \rightarrow \infty) \propto a^{-4}$$



Casimir Energy vs. Vacuum Energy

- Casimir energy: $p_x = -\rho$, $p_y > 0$.
 p_y cannot be tuned away.
- Conventional vacuum energy (brane tension):
 $p_x = -\rho$, $p_y = 0$.
 p_y can in principle be tuned away.



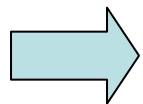
Strategy for the Smallness of Dark Energy

Supersymmetry

Brane World

$$\rho_{\text{vac}} = \underbrace{\text{[blue bar]}}_{\text{boson}} + \underbrace{\left(- \text{[yellow bar]} \right)}_{\text{fermion}} = 0$$

BUT ~~SUSY~~ only on the brane



$$\rho_{\text{vac}}^{(4)} \sim (m_n - m_{n-1}) \Delta m_n^{-2}$$

- It can be shown that this will induce a KK gravitino mass-shift for the n th mode

$$\delta m_n \equiv g_5 \langle \Phi \rangle \int \frac{dy}{a} \delta(y - \pi) \sqrt{-g} \frac{1}{2\pi a} [e^{2\sigma} f^{(n)}]^2$$

- Graviton, on the other hand, remains massless (at tree-level).

- On the TeV brane,

$$\begin{aligned}\lim_{z \rightarrow \infty} J_\nu(z) &= \sqrt{\frac{2}{\pi z}} \cos\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) \\ &\simeq \sqrt{\frac{2}{n\pi}} \cos[(n-1)\pi] = \pm \sqrt{\frac{2}{n\pi}}, \\ \lim_{z \rightarrow \infty} Y_\nu(z) &= \sqrt{\frac{2}{\pi z}} \sin\left(z - \frac{\nu\pi}{2} - \frac{\pi}{4}\right) \\ &\simeq \sqrt{\frac{2}{n\pi}} \sin[(n-1)\pi] = 0.\end{aligned}$$

- The normalization constant is

$$N_n \simeq \frac{\alpha_G^{-1}}{\sqrt{2\pi k a}} J_1(\alpha_G^{-1} m_n/k) \simeq \frac{\alpha_G^{-1}}{\sqrt{n\pi^2 k a}}.$$

- Therefore on the TeV brane

$$f^{(n)}(y = \pi) \simeq \sqrt{2\pi k a} \alpha_G^{1/2}.$$

- By definition, the Casimir energy under SUSY breaking is

$$\Delta\rho_C^{(4)} = \rho_C^{(4)} \Big|_{\delta m_n^2} - \rho_C^{(4)} \Big|_{\delta m_n^2=0} \forall n ,$$

which is

$$\Delta\rho_C^{(4)} \sim (m_n - m_{n-1})^2 \delta m_n^2 .$$

The mass shift can be show nto scale as

$$\delta m_n \sim \alpha_G^3 M_{\text{Pl}} .$$

Summary

- Dark energy may very well be a cosmological constant.
- The numerical coincidence between the SM-Planck and the SM-DE hierarchies implies a deeper connection between the two.
- If true, the well-known smallness and concordance problems of CC can be understood.
- Casimir energy in a SUSY-breaking brane world may provide the key to the solution of DE problem.
- It is remarkable that, guided by our Ansatz, we manage to construct a model for CC whose energy scale is indeed suppressed by 2 powers of the “gravity fine structure constant” (the hierarchy)!