PHASE SPACE CONSTRAINTS ON NEUTRINO LUMINOSITIES

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Approach

- Introduction
- General Case
- General Relativistic Case
- Special Relativistic Case
- Modeling GRB's

Introduction

- Most energetic events accompanied by
  - electromagnetic radiation
  - neutrino emission
- Typical example: Gamma Ray Bursts
- Phase space constraints

General Case

- Neutrino parameters:

  - E_ν = Average energy of the neutrino’s from source
  - n_ν = number density of the emitted neutrinos produced
  - Phase space constraint condition:

$$d^3x.d^3p \geq \hbar^3$$

General Case (Contd.)

- The constraints are:

$$n_\nu^{-1}\left(\frac{E_\nu}{c}\right)^3 \geq \hbar^3 \Rightarrow n_\nu \leq \left(\frac{E_\nu}{\hbar c}\right)^3 ; \varepsilon_\nu = \left(\frac{E_\nu}{c}\right)^3 \frac{1}{\hbar^3} E_\nu = \frac{E_\nu^4}{(\hbar c)^3}$$

- Equation (2) implies that:

$$\left(\varepsilon_\nu = n_\nu E_\nu\right)$$

$$f_\nu \leq \frac{E_\nu^4}{4c^2}\frac{1}{\hbar^3}$$

- For, $E_\nu \approx 10 MeV$; this implies:
- An E_ν^4 energy dependence!
- For 1MeV, this is of the order of:

$$f_\nu \leq 3\times10^{39} ergs/cm^2/s$$

General Case (Contd.)

- Generalization using diffusion time t_d, neutrino opacity κ_ν, and neutrino number density *n*, we get:

$$t_d \approx \frac{1}{n\sigma_\nu c} \; ; \; \kappa_\nu \sim n m_P \sigma_\nu$$

- Using (3),(5) and a total neutrino opacity {Ref.[8]}, we have:

$$\kappa_\nu = 2\times10^{-17}\,\rho\left({kT_\nu}\big/{4MeV} \right)^2 cm^{-1}$$

General Case (Contd.)

- If the number of neutrino scatterings is *N* ;

$$L_\nu \approx 4\pi\left(N^{1/2} n^{-1/2} \sigma_\nu^{-1/2}\right)^2 \frac{E_\nu^4}{4c^2}\frac{1}{\hbar^3} \approx \frac{4\pi N}{\sigma_\nu n}\frac{E_\nu^4}{4c^2\hbar^3} \approx \frac{\pi N E_\nu^4}{\sigma_\nu n c^2 \hbar^3}$$

- Plugging in the values, we have:

$$L_\nu \approx 2\times 10^{52}\, ergs/\sec$$

General Relativistic Case

- Including General Relativistic Effects:

$$\left(\sqrt{g_{00}}\,d^3x\right)\left(\sqrt{g_{00}}\,d^3p\right) \geq \hbar^3$$

- Schwarzschild metric coefficients:

$$g_{00} = \left(1 - \frac{2GM}{rc^2}\right);\ g_{11} = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)};\ g_{22} = r^2;\ g_{33} = r^2 \sin^2\theta$$

- Number density restricted by the phase space constraint:

$$n_\nu \leq \left(1 - \frac{2GM}{rc^2}\right)\left(\frac{E_\nu}{\hbar c}\right)^3$$

- Reduction in Energy due to red-shift is given by:

$$\frac{\Delta E}{E} = \frac{GM}{rc^2}$$

General Relativistic Case (Contd.)

- Consider the case:

  $M = 1.4 M_{\Theta}; r = 10 Km$

  Energy shift of the order 0.1E;

- Upper limit for Energy Density & maximum flux:

  $\varepsilon_{\nu} \leq \left(1 - \frac{2GM}{rc^2}\right) \frac{E_{\nu}^4}{(\hbar c)^3}$ and $F \leq \frac{c}{4} \varepsilon_{\nu}$

- For $E_{\nu} \approx 10 MeV$

  $f_{\nu} \leq 5.5 \times 10^{38} ergs / cm^2 / s$

Special Relativistic Case

- For a neutrino of energy E_ν, number density *n:*

$$d^3x = R_D^3 = \left(\frac{3}{4\pi}\frac{E_\nu}{\Gamma^2 n m_P c^2}\right) \textit{and}\; d^3p = \left(\frac{E_\nu}{\Gamma c}\right)$$

- Phase space constraint:

$$\left(\frac{3}{4\pi}\frac{E_\nu}{\Gamma^2 n m_P c^2}\right)\left(\frac{E_\nu}{\Gamma c}\right)^3 \geq \hbar^3$$

- The number density: $n \leq \left(\frac{3}{4\pi}\frac{E_\nu}{\Gamma^2 m_P c^2}\right)\left(\frac{E_\nu}{\hbar^3 \Gamma c}\right)^3$

Special Relativistic Case (Contd.)

- Limit on Energy Density:

$$\varepsilon_\nu \leq \left(\frac{3}{4\pi} \frac{E_\nu}{\Gamma^2 m_P c^2} \right) \left(\frac{E_\nu}{\hbar^3 \Gamma c} \right)^3 E_\nu$$

- Notice the E_ν^5 dependence and also the sharp Γ^5 dependence!!
- Phase space constraint on the flux: $f_\nu \leq \frac{c}{4} \varepsilon_\nu$
- For $E_\nu \approx 10 MeV$

$$f_\nu \leq 4 \times 10^{36} ergs / cm^2 / s$$

Special Relativistic Case (Contd.)

- Neutrinos: F-D statistics
- Maximum allowed Energy is:

  E_F = Fermi Energy

- Energy Flux for Neutrino Emission in a cone of angle θ_ν is given by:

$$F_\nu = \int_0^{\theta_\nu} d\Omega \int_0^{E_F} cE_P^2 \frac{dp}{\hbar^3} = \frac{1-\cos\theta}{16\pi^3} c\varepsilon_\nu$$

- Energy Density is given by: $\varepsilon_\nu = E_F \left(p_F / \hbar \right)^3$

Special Relativistic Case (Contd.)

- Giving upper limit on neutrino luminosity:

$$4\pi R_0^2 F_\nu = R_0^2 \frac{\Gamma^2}{2} c\varepsilon_\nu \; ; R\,\Gamma^2 \big/ 2 \le c\Delta t$$

- Bursts energy: $$E_\nu < L_\nu \Delta t \frac{\Gamma^2}{2}$$

- If neutrino burst energy is constrained by binding energy of the neutron star then the burst duration: $$\Delta t = \frac{2E_\nu}{\Gamma^2 L_\nu}$$

Special Relativistic Case (Contd.)

- Including special and general relativistic effects, neutrino flux is:

$$f_\nu \le \frac{c}{4}\left(1-\frac{2GM}{rc^2}\right)\left(\frac{3}{4\pi}\frac{E_\nu}{\Gamma^2 m_P c^2}\right)\left(\frac{E_\nu}{\hbar^3\Gamma c}\right)^3 E_\nu$$

- Comoving luminosity: $L_\nu = f_\nu 4\pi R^2$
- In the observer's frame:

$$L_\nu = f_\nu 4\pi R^2\Gamma^2 = \frac{c}{4}\left(1-\frac{2GM}{rc^2}\right)\left(\frac{3E_\nu}{\Gamma^2 m_P c^2}\right)\left(\frac{E_\nu}{\hbar^3\Gamma c}\right)^3 E_\nu R^2\Gamma^2$$

Modelling GRB

- Pair neutrino annihilation:

$$\nu_e + \overline{\nu}_e \rightarrow e^+ + e^- + \gamma + \gamma$$

- Minimum energy required:

$$2m_e c^2 \approx 1MeV$$

- Cross section for the interaction:

$$R = \frac{G_F^2 E^2}{(\hbar c)^4} n_e \times n_e c \quad \text{where} \quad R = \sigma v n^2$$

Modelling GRB

- Energy density: $\varepsilon = R(kT)$

$$\varepsilon = \frac{G_F^2 E^2}{(\hbar c)^4}\left(\frac{kT}{\hbar c}\right)^3 \times \left(\frac{kT}{\hbar c}\right)^3 c(kT) = 10^{-56}\left(\frac{T}{1K}\right)^9 ergs/cm^3$$

- The flux is given by:

$$f = \frac{c}{4}\varepsilon = 7.5\times 10^{-47}\left(\frac{T}{1K}\right)^9 ergs/cm^2/s$$

- Intense Gamma flux due to neutrino annihilation:

$$\nu + \overline{\nu} \rightarrow \gamma + \gamma$$

Modelling GRB

- Hadron interaction: high energy neutrinos + high energy gamma rays

$$p + p \rightarrow p + p + \pi^0$$

$$\pi^0 \rightarrow 2\gamma$$

$$n \rightarrow p + e^- + \overline{\nu}_e$$

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_\mu + \nu_e$$

- Phase space constraints applied to high energy neutrinos, gives a maximum released neutrino energy:

$$\frac{G_F^2 E_t^2 E_\nu^4}{\hbar^7 c^6} = \left(\frac{G_F E_t}{\hbar^2 c^2} \right)^2 \frac{c\varepsilon_\nu}{\Gamma^4} ergs/\sec$$

Modelling GRB

- For example: For a pion energy $E_\pi = 200 GeV$
- Decay neutrino energy:

$$E_\nu = \left(\frac{m_\pi^2 - m_\mu^2}{m_\pi^2} \right) E_\pi \approx 85 GeV$$

- Upper limit for neutrino momentum:

$$P_\nu = \Gamma(1 + \beta) E_\nu / c \approx 2 \times 10^5 \, GeV/c = 2 \times 10^2 \, TeV/c$$

For $\beta \approx 1; (v \approx c)$ and $\Gamma = 1000$

Modification in Phase Space due to Generalized uncertainty Principle (GUP)

- $\left[x, y\right] = iL_{pl}^2 , \left[x, p_x\right] = i\hbar$, …
- A typical GUP relation [20]:

$$\Delta x \Delta p \geq \hbar + \frac{\lambda}{\hbar}\left(\Delta p\right)^2$$

- Where, λ has dimensions of L_{pl}^2 , where, L_{pl} is the Planck Length and, ;

$L_{pl}^2 \approx 10^{-66} cm^2 \approx$ 1 attoshed;1 shed $= 10^{-24}$ barns

GUP (Contd.)

- Phase space is modified from: $\frac{dVd^3p}{(2\pi\hbar)^3}\left(\Delta x\Delta p \sim 2\pi\hbar\right)$
- To

$$\frac{dVd^3p}{(2\pi\hbar)^3\left(1+\lambda p^2\right)^3}, p^2 = p_i p^i, i = 1,2,3$$

- The non-commutativity becomes: $\left[\hat{x}, \hat{p}\right] = i\hbar\left(1+\lambda p^2\right)$
- Modification of $\hbar$ to

$$\hbar' = \hbar\left(1+\lambda p^2\right)$$

- {first Suggested in ref [21]}

GUP (Contd.)

- Above relation suggests a modification to the phase space constraint:

$$d^3x.d^3p \geq \hbar^3$$

- The GUP (eq.16) implies a phase space constraint of the form:

$$d^3x.d^3p \geq \hbar^3\left(1+\lambda p^2\right)^3$$

GUP (Contd.)

- Number Density of neutrino's takes the shape:

$$n_\nu^{-1}\left(\frac{E_\nu}{c}\right)^3 \geq \hbar^3\left(1+\lambda p^2\right)^3$$

- As $\lambda p^2 << 1$, expand the R.H.S above binomially:

$$n_\nu^{-1}\left(\frac{E_\nu}{c}\right)^3 \geq \hbar^3\left(1+3\lambda p^2+\ldots\right)$$

(Neglecting higher order terms in λp^2)

GUP (Contd.)

- Giving upper limit on number density as:

$$n_\nu \leq \left(\frac{E_\nu}{\hbar c}\right)^3 \left(1 + 3\lambda p^2\right)^{-1}$$

- Giving a phase space constraint on neutrino flux (compared to (3))

$$f_\nu \leq \frac{E_\nu^4}{4c^2\hbar^3}\left(1 - \frac{3\lambda}{\hbar^2}\left(\frac{E_\nu}{c}\right)^2\right)$$

Conclusions

- For neutrino energies of the order of TeV, and $\lambda \approx L_{pl}^2 \approx 10^{-66} cm^2$, the modification is of the order of 10^{-32}
- Effects are too small to be detected.
- In the weak interaction scale, i.e., $\lambda \approx l_W^2 \approx 10^{-33} cm^2$ for TeV energies, correction is of the order 10. (large modification)
- Assumed reduction in the scale does not take place.

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