The QCD Transition: Lattice Results

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3. The transition temperature

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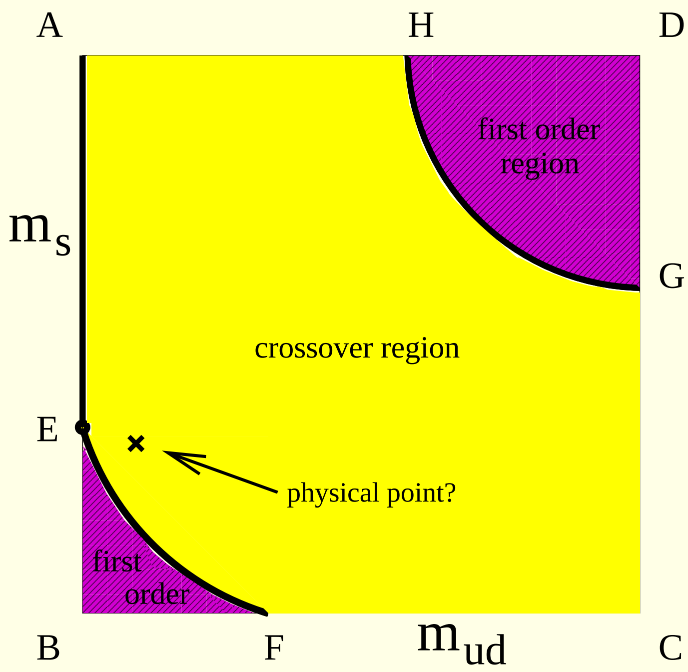
Phase diagram



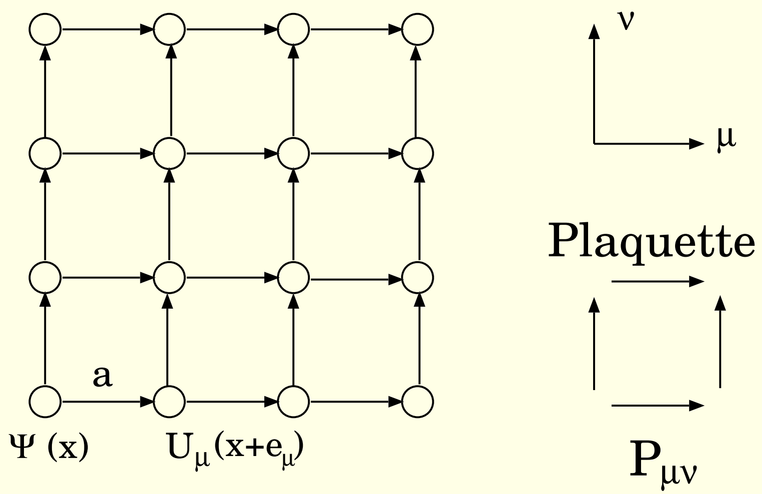
$m_{ud}, m_s \to \infty$: pure SU(3) theory, 1st order phase transition
$m_{ud} = 0, m_s \to \infty$: 2 flavor massless QCD, 2nd order transition
$m_{ud} = m_s = 0$: 3 flavor massless QCD, 1st order phase transition

Physical point: probably in the cross-over region

Questions:

Is the transition really a crossover?
What is the transition temperature?

Lattice QCD introduction



Fundamental Fields:

Gauge fields:

$U_\mu(x) \in SU(3)$ live on the links

Quarks:

$\Psi(x), \bar{\Psi}(x)$

anti-commuting Grassmann variables live on the sites

Wilson fermions: $\mathcal{O}(a)$ artefacts

Staggered fermions: O(a^2), BUT flavour symmetry violation

Partition function

$$Z = \int dU d\Psi d\overline{\Psi} e^{-S_E}$$

- S_E is the Euclidean action
- Parameters:
  - gauge coupling g
  - quark masses m_i ($i = 1..N_f$)
  - (Chemical potentials μ_i)
  - Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

$$T = \frac{1}{N_t a}$$

Continuum limit: $a \to 0$

- Renormalization: keep the physical spectrum constant
- at finite T:
  - continuum limit $\Longleftrightarrow N_t \to \infty$

The order of the cosmological QCD transition

Y. Aoki, G. Endrodi, Z. Fodor, SDK, K.K. Szabo, Nature 443 (2006) 675

Lattice action:

Symanzik improved gauge, stout improved fermionic action

In the early universe: $\mu \ll m_N$ $\longrightarrow$ we used $\mu = 0$

Main steps of the analysis:

finite temperature:

- physical quark masses
- N_t=4,6,8,10 lattices
- correspond to $a \approx$ 0.28 fm, 0.19 fm, 0.14 fm, 0.11 fm

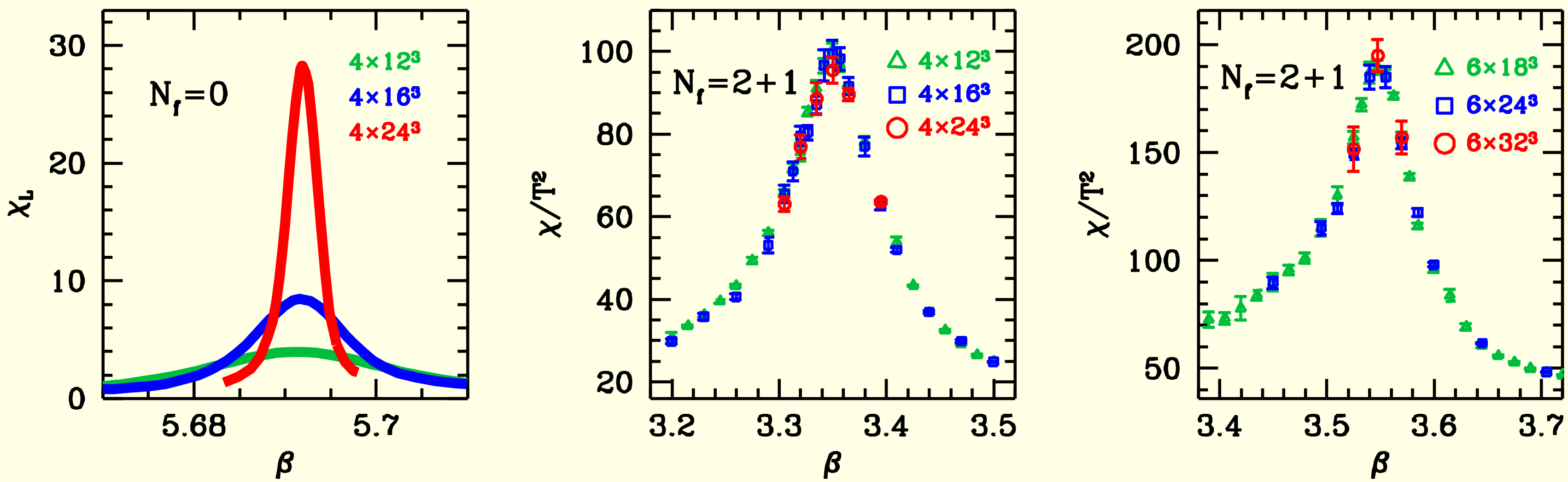
zero temperature:

- renormalization
- chiral extrapolation using larger masses ($3 - 10 \cdot m_{ud}$)

Finite size scaling of the transition

Chiral susceptibilities: χ=(T/V)∂^2logZ/∂m^2

first order transition $\Longrightarrow$ peak width $\propto 1/V$, peak height $\propto$ V
second order transition $\Longrightarrow$ width $\propto 1/V^{\alpha}$, height $\propto 1/V^{\beta}$
cross-over $\Longrightarrow$ peak width $\approx$ constant, peak height $\approx$ constant

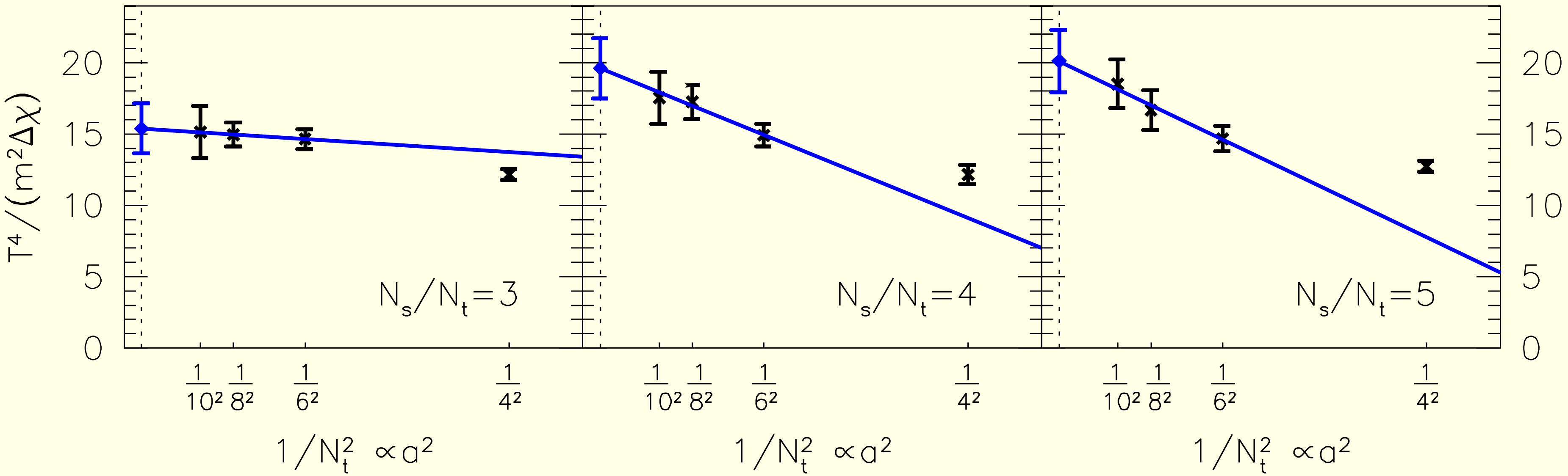


for aspect ratios 3–6 (an order of magnitude larger volumes)
volume independent scaling $\Longrightarrow$ cross-over

do we get the same result (cross-over) in the continuum limit?

We need a continuum extrapolation (width, height)

calculate $m^2\Delta\chi = m^2[\chi(T\neq 0)-\chi(T=0)]$ at the transition point

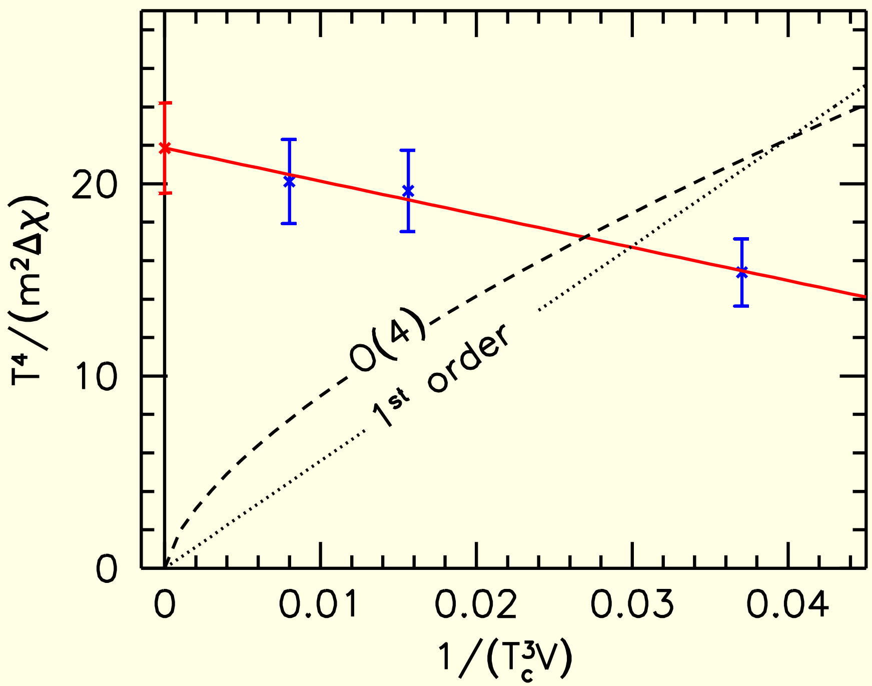


continuum limit values are obtained from N_t=4,6,8,10

N_t=6,8,10 temporal sizes are already in the a^2 scaling region

the three continuum extrapolated values do not show $1/V$ scaling

Finite size scaling analysis with continuum extrapolated $m^2\Delta\chi$



the result is consistent with an approximately constant behavior
for a factor of 5 difference within the volume range

chance probability for $1/V$ is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

the QCD transition at μ=0 is a cross-over

The transition temperature

Y. Aoki, Z. Fodor, SDK, K.K. Szabo, Phys. Lett. B643 (2006) 46-54

$T = 0$:

set the physical scale and locate the physical point

Three quantities are needed (m_π and m_K for the quark masses)

Several possibilities for the third quantity

- string tension (not existing in full QCD)
- static quark potential at intermediate distances (r_0)
- directly measurable quantities (e.g. f_K)

Further quantities are predictions (e.g. r_0, f_π, m_{K^*})

$T > 0$:

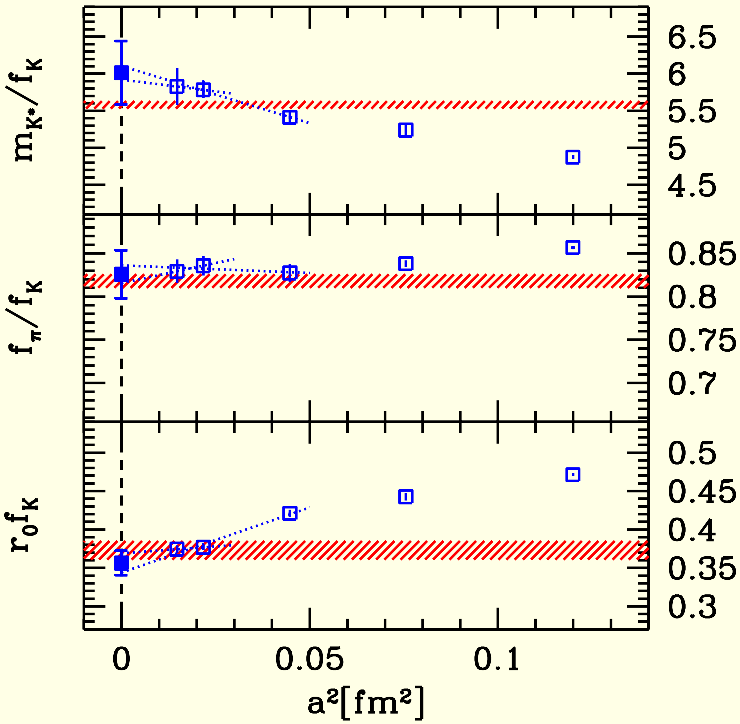
cross-over $\longrightarrow$ different definitions give different T_c

Possible choices:

- Chiral susceptibility
- Quark number susceptibility
- Polyakov-loop

T=0 Simulations

- m_π, m_K and f_K was used to set the quark masses and scale
- $m_{ud} \approx 3,5,7,9 \times m_{ud,phys}$ together with chiral extrapolation
- lattices from $12^3 \cdot 24$ up to $24^3 \cdot 32$



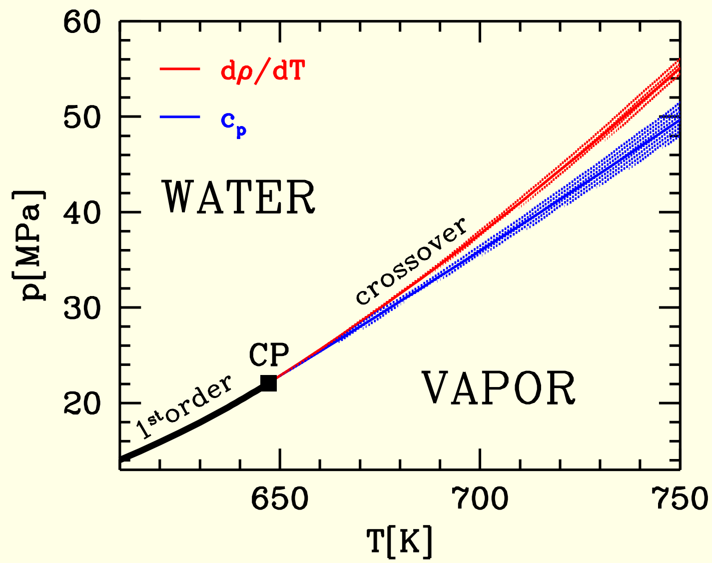
Predictions for m_{K^*}, f_π and consistent with experimental values

r_0 is consistent with MILC measurement

T>0 Simulations

No unique T_c

Example of water-steam transition

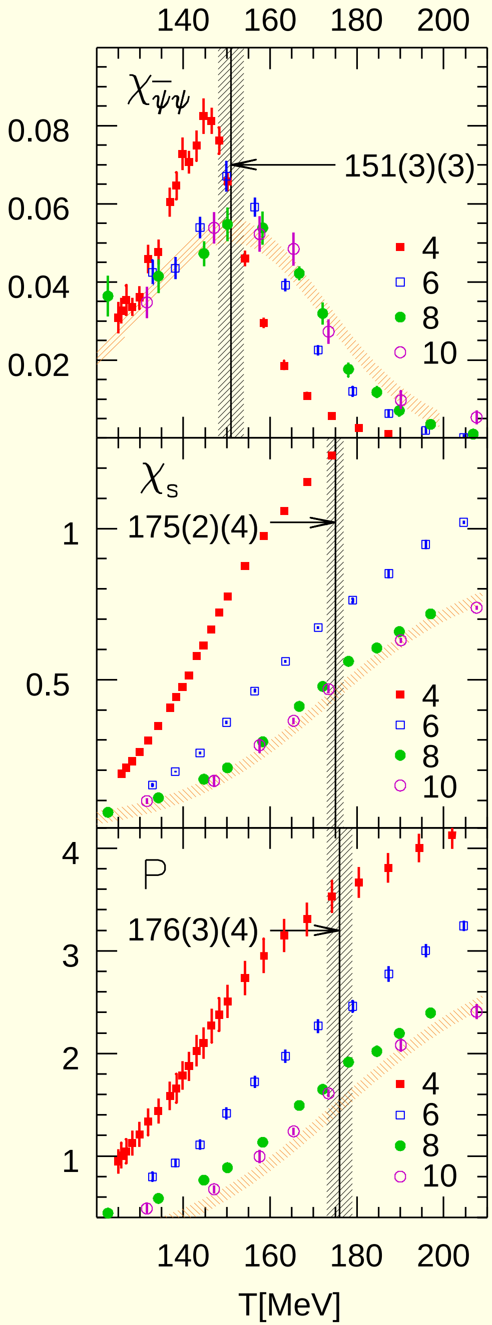


above the critical point c_p and $d\rho/dT$ give different T_c's.

Our choices in QCD

$\frac{m^2\Delta\chi}{T^4}$ $\longrightarrow$ chiral transition

Quark number susceptibility
Polyakov loop $\longrightarrow$ de-confinement transition



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

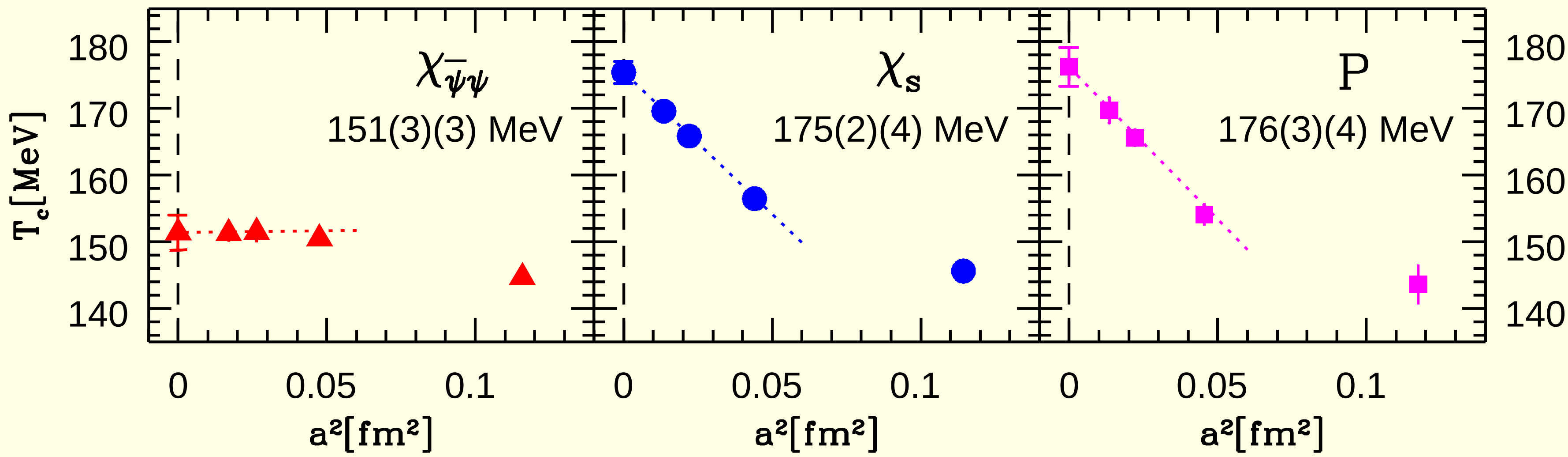
ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

ΔT_c=38(5)(1) MeV

Continuum extrapolations



$N_t = 4, 6, 8$ show nice scaling for all quantities

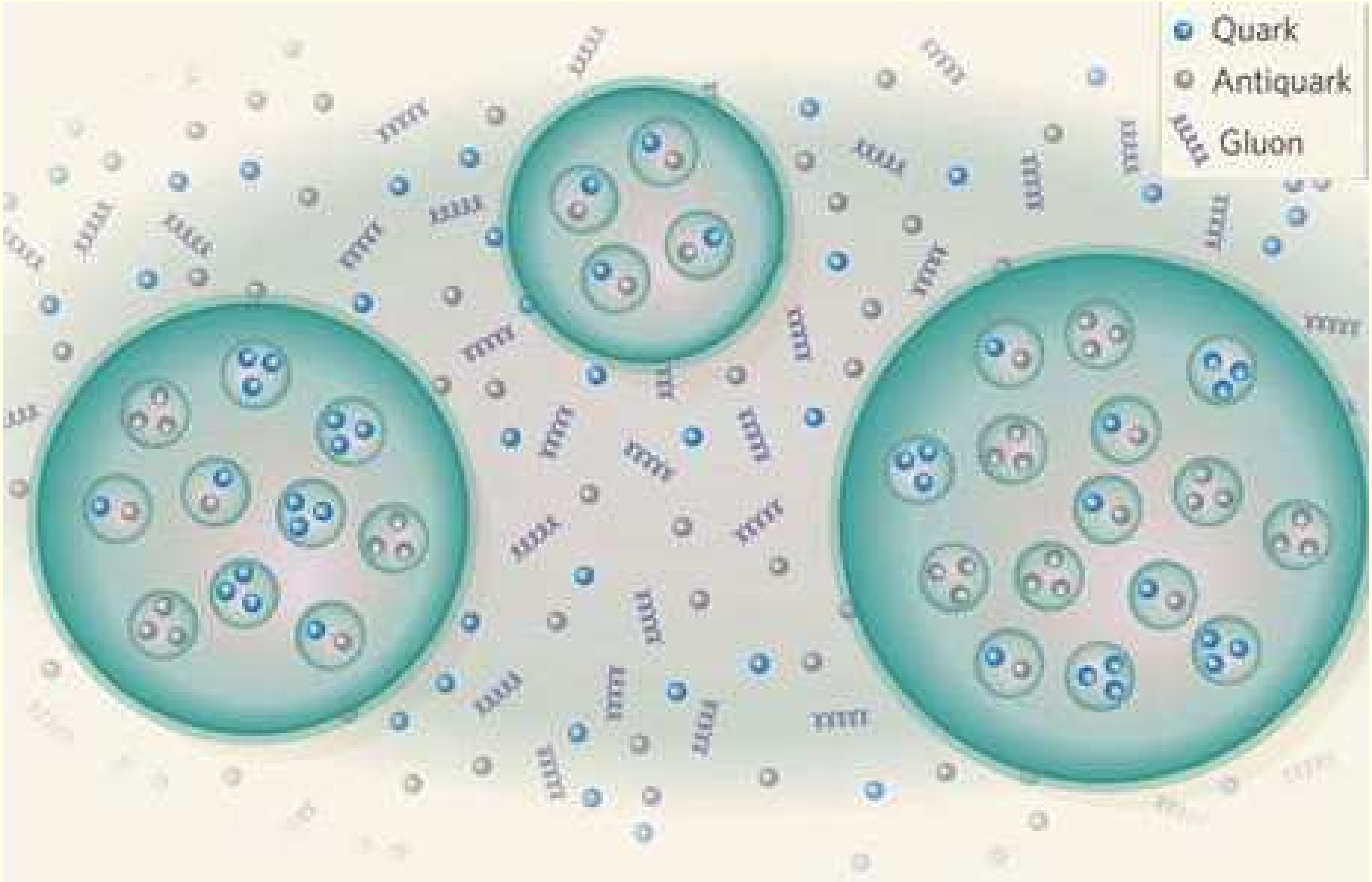
Chiral and de-confinement transitions at different locations
25(4) MeV difference

Note: different normalization leads to different T_c
(e.g. $\Delta\chi/T^2$ leads to ≈10 MeV higher T_c)

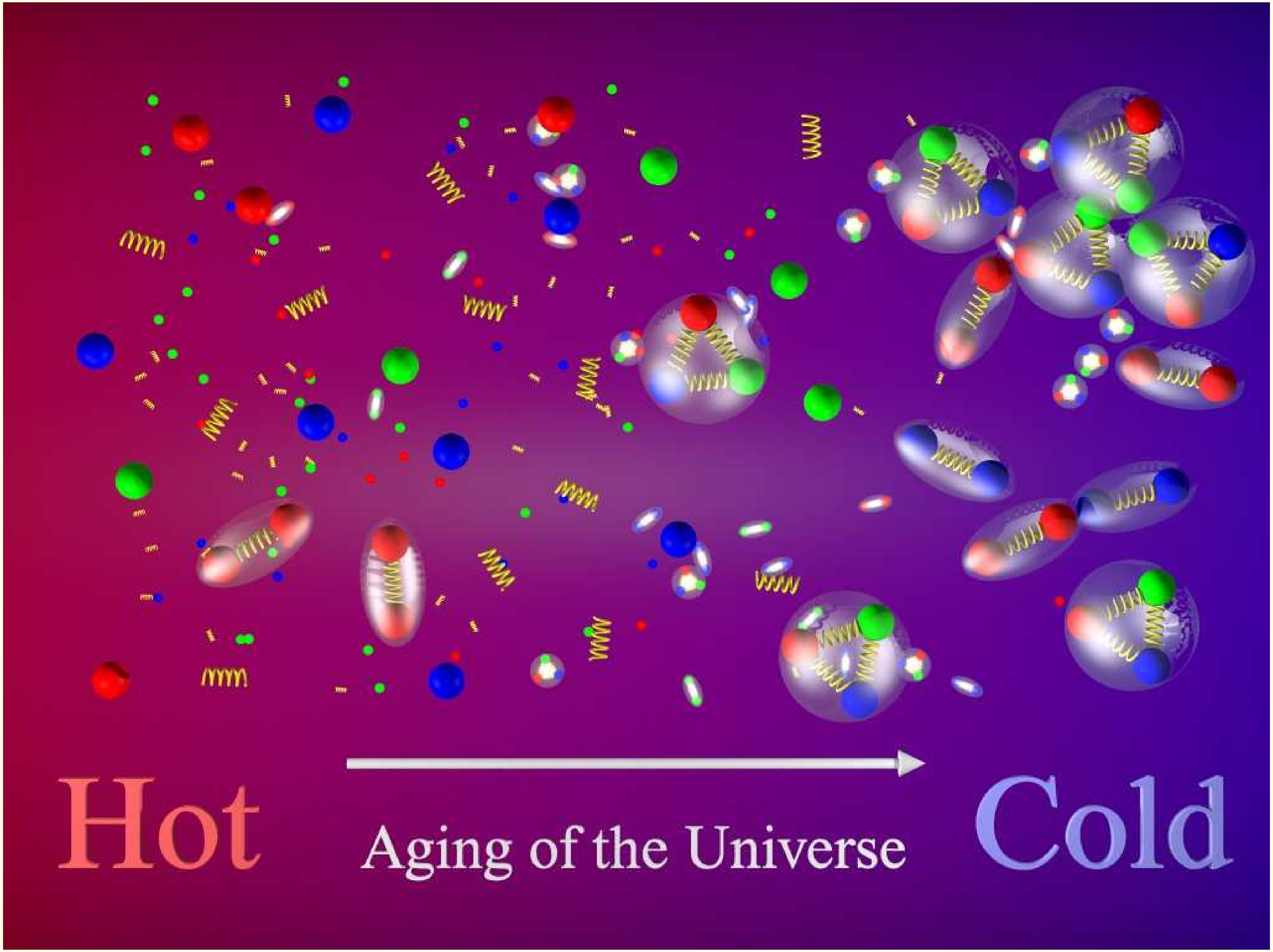
Summary

- finite size scaling analysis of the chiral susceptibility
- continuum limit using $N_t = 4, 6, 8, 10$
- no volume dependence, inconsistent with 1^{st} or 2^{nd} order phase transition
- The QCD transition is a cross-over

This could have happened:



But this did happen:



- The transition temperature is determined

- Chiral susceptibility:

  T_c=151(3)(3) MeV, ΔT_c=28(5)(1) MeV

- Quark number susceptibility:

  T_c=175(2)(4) MeV, ΔT_c=42(4)(1) MeV

- Polyakov loop:

  T_c=176(2)(4) MeV, ΔT_c=38(5)(1) MeV