



# Palatini $f(R)$ Cosmology



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# Outline

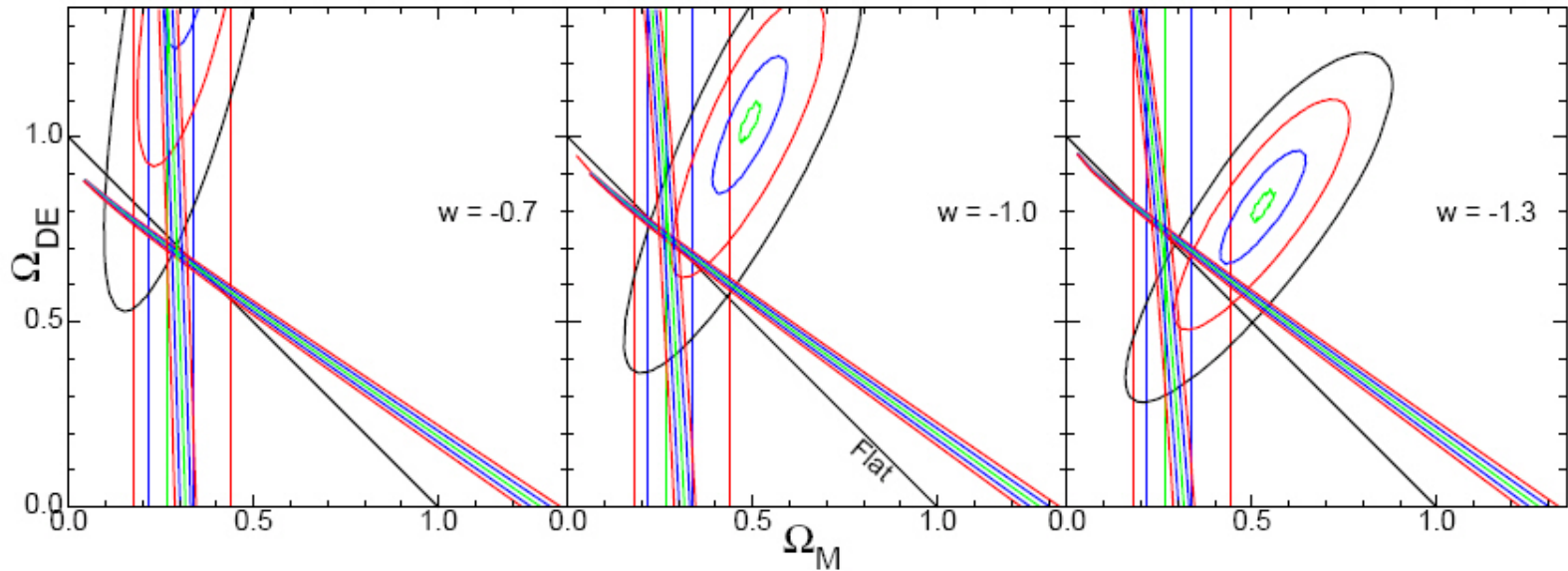


- ❖ Experimental Evidences
- ❖ Motivations
- ❖ Models
- ❖ Palatini  $f(R)$  gravity
- ❖ Phenomena (Evolution of Background, Growth factors)
- ❖ Stability of Palatini  $f(R)$  gravity
- ❖ Summary



# Experimental Evidences

## (Cosmological Concordance ?!)



- ❖ SNe Ia (Fig from E . L. Wright 06)
- ❖ CMB
- ❖ Values for H (Age of Universe)
- ❖ Baryonic Oscillations  $\rightarrow$  SDSS



# Motivations



$$\delta G_{\mu\nu}(\text{MG}) + R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} + 8\pi G T_{\mu\nu}(\text{DE})$$

$$S = \int d^n x \sqrt{-g} \left[ \frac{1}{2}f(R, \phi) + \mathcal{L}_\phi(g_{\mu\nu}, \phi, \partial\phi) + \mathcal{L}_m(g_{\mu\nu}, \Psi) \right]$$

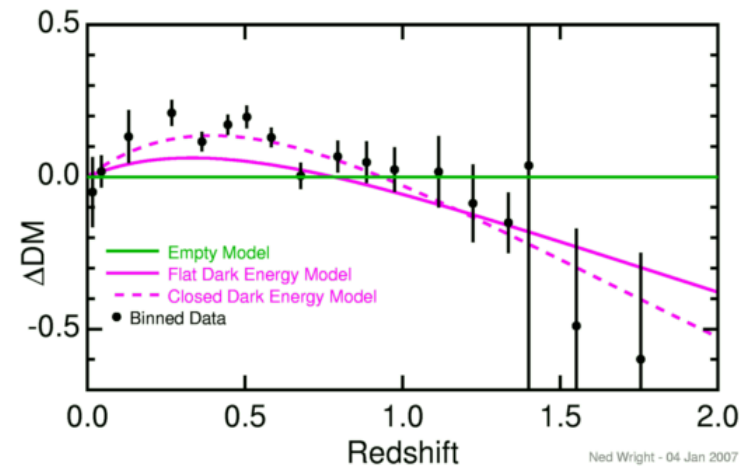
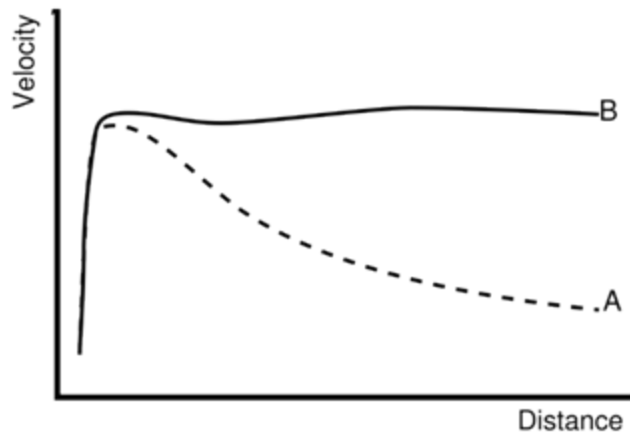
- ❖ Observations are **in**consistent with the expected universe (GR[FLRW] + Matter). Based on **homogeneous** and **isotropic** space-time.
- ❖ Need to change either **G**ravity or **M**atter (if we still believe homogeneity and isotropy).
- ❖ Heard from D.Huterer, P Zhang, B. Wang



# Motivations



- ❖ To explain galaxy rotational curves (1959)
  - : Dark Matter  $\leftrightarrow$  **M**ODIFIED **N**EWTONIAN **D**YNAMICS (1983)
- ❖ To explain accelerating universe (1998)
  - : Dark Energy  $\leftrightarrow$   $f(R)$





# Generalized Gravity Theories I



- ❖ A broad class of alternative gravity theories
- ❖  $\Psi$  : matter fields
- ❖  $\Phi$  : a scalar field

$$S = \int d^n x \sqrt{-g} \left[ \frac{1}{2} f(R, \phi) + \mathcal{L}_\phi(g_{\mu\nu}, \phi, \partial\phi) + \mathcal{L}_m(g_{\mu\nu}, \Psi) \right]$$

$$\mathcal{L}_\phi = -\frac{M^2}{2} \omega(\phi) (\partial\phi)^2 - V(\phi)$$

$$(\partial\phi)^2 = \nabla_\mu \phi \nabla^\mu \phi. \quad F(R, \phi) = \partial f(R, \phi) / \partial R.$$



# Generalized Gravity Theories II



Generalized gravity	$\frac{1}{2}f(R, \phi)$	$\mathcal{L}_\phi(\phi, \partial\phi)$	$p(R, \phi)$	$\varphi$	$\tilde{V}(\varphi)$	Ref
Nonlinear gravity	$\frac{1}{2}f(R)$	$\omega = 0, V = 0$	$p = F(R)$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$	[1]
$R^2$ -gravity	$\frac{1}{2}(R + \alpha R^2)$	$\omega = 0, V = 0$	$p = 1 + 2\alpha R$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$	[2]
$1/R$ -gravity	$\frac{1}{2}(R - \mu^4/R)$	$\omega = 0, V = 0$	$p = 1 + \mu^4/R^2$	$\sqrt{\frac{3}{2}} \ln F$	$\frac{FR-f}{2F^2}$	[3]
Scalar-tensor theory	$\frac{1}{2}F(\phi)R$	$\omega(\phi), V(\phi)$	$p = F(\phi)$	$\int \sqrt{\frac{\omega}{F} + \frac{3}{2} \frac{F'^2}{F^2}} d\phi$	$\frac{V}{F^2}$	[4]
Brans-Dicke theory	$\phi R$	$\omega(\phi) = 2\frac{\omega}{\phi}, V = 0$	$p = \phi$	$\int \sqrt{\frac{\omega}{F} + \frac{3}{2} \frac{F'^2}{F^2}} d\phi$	0	[5]
Dilaton	$\frac{1}{2}e^{-\phi} R$	$\omega(\phi) = e^{-\phi}, V = 0$	$p = e^{-\phi}$	$\frac{5}{2}\phi$	0	[6]
NMC scalar	$\frac{1}{2}(1 + \xi\phi^2)R$	$\omega = 1, V(\phi)$	$p = 1 + \xi\phi^2$	$\int \frac{\sqrt{1 + \xi(6\xi - 1)\phi^2}}{1 - \xi\phi^2} d\phi$	$\frac{V}{1 - \xi\phi^2}$	[7]
CC ( $\xi = \frac{1}{6}$ )	$\frac{1}{2}(1 + \frac{1}{6}\phi^2)R$	$\omega = 1, V(\phi)$	$p = 1 + \frac{1}{6}\phi^2$	$\sqrt{6} \tanh^{-1} \frac{\phi}{\sqrt{6}}$	$\frac{V}{1 - \frac{1}{6}\phi^2}$	[8]
Induced Gravity	$\frac{1}{2}\epsilon\phi^2 R$	$\omega = 1, V(\phi)$	$p = \epsilon\phi^2$	$\sqrt{6 + \frac{1}{\epsilon}} \ln \phi$	$\frac{V}{\epsilon\phi^2}$	[9]
GR with a scalar	$\frac{1}{2}R$	$\omega = 1, V(\phi)$	$p = 1$	$\phi$	$V$	



# Palatini f(R) Gravity



- ❖ Connection  $\Gamma_{\mu\nu}^{\lambda}$  defines parallel transport (covariant derivative)
- ❖ Riemann tensor  $R_{\nu\sigma\lambda}^{\mu} = \nabla_{\sigma}\Gamma_{\nu\lambda}^{\mu} - \nabla_{\lambda}\Gamma_{\nu\sigma}^{\mu} + \Gamma_{\alpha\sigma}^{\mu}\Gamma_{\nu\lambda}^{\alpha} - \Gamma_{\alpha\lambda}^{\mu}\Gamma_{\nu\sigma}^{\alpha}$
- ❖ In standard GR, we assume  $\nabla_{\lambda}g_{\mu\nu} = 0$
- ❖ Levi-Civita connection  $\Gamma_{\mu\nu}^{\lambda} = \frac{1}{2}g^{\lambda\sigma}(\partial_{\mu}g_{\nu\sigma} + \partial_{\nu}g_{\mu\sigma} - \partial_{\sigma}g_{\mu\nu})$



# Palatini f(R) Gravity



❖ Action in GR  $S_{\text{GR}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R(g) + S_{\text{M}}(g_{\mu\nu}, \psi)$

❖ Action in metric-affine f(R) theories of gravity

$$S_{\text{MA}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(\hat{R}) + S_{\text{M}}(g_{\mu\nu}, \Gamma_{\mu\nu}^{\lambda}, \psi)$$

❖ Action in Palatini f(R) theories of gravity

$$S_{\text{Pal}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(\hat{R}) + S_{\text{M}}(g_{\mu\nu}, \psi)$$

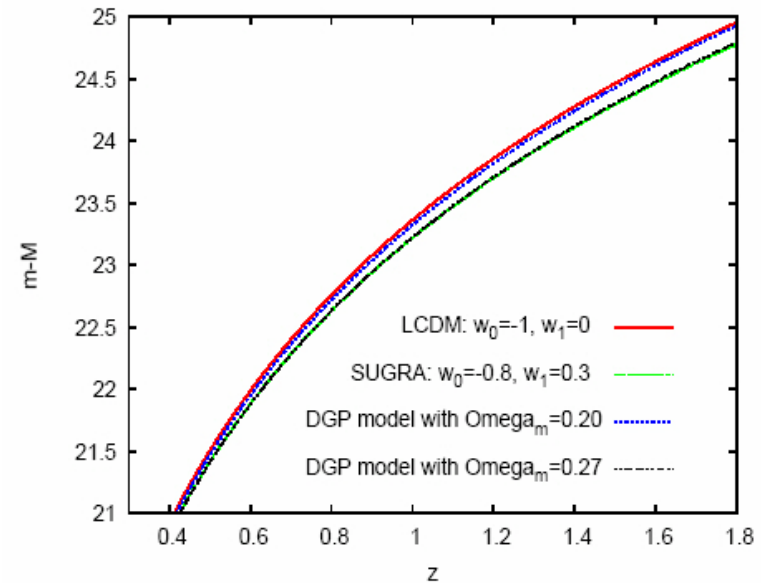


# Background Evolution



$$H^2 - \delta H^2 = \sum_{i=\text{rad,mat}} \frac{8\pi G}{3} \rho_i$$

$$\omega_{\text{DE}} = -1 + \frac{1}{3} \frac{d \ln \delta H^2}{d \ln(1+z)}$$



- ❖ SNe Hubble diagrams for DE and DGP models (Fig from M.Ishak et al, 06)

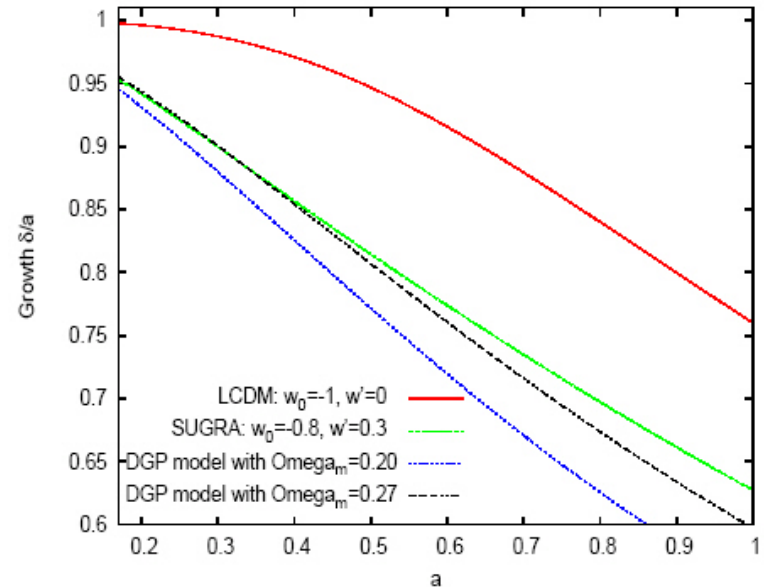


# Growth Factors



$$\ddot{\delta} + 2H_{DGP}\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0$$

$$\beta = 1 - 2r_c H_{DGP} \left(1 + \frac{\dot{H}_{DGP}}{3H_{DGP}^2}\right)$$



## ❖ Growth factors of linear perturbations



# Stability of Palatini f(R) Gravity



$\theta$  the deviation from  $\zeta$  conservation

$\epsilon$  the deviation from the superhorizon metric

$$\zeta' = \Phi' + \Psi - H'q = -\frac{H'}{H} \left( \frac{k}{aH} \right)^2 B\theta$$

$$\Phi'' + \Psi' - \frac{H''}{H'}\Phi' + \left( \frac{H'}{H} - \frac{H''}{H'} \right) \Psi = -\left( \frac{k}{aH} \right)^2 B\epsilon$$

$$B = \frac{F'}{F} \frac{H}{H'} \quad \text{where} \quad F = \frac{\partial f(\hat{R})}{\partial \hat{R}}$$



# Stability in a model (SL 0710.2395)



$$f(\hat{R}) = \beta \hat{R}^n$$

$$\frac{H'}{H} = -\frac{3}{2n}, \quad \frac{H''}{H} = \left(-\frac{3}{2n}\right)^2$$

$$\frac{F'}{F} = \frac{F''}{F'} = \frac{3(1-n)}{n}, \quad \frac{F''}{F} = \frac{F'''}{F''} = \left(\frac{3(1-n)}{n}\right)^2$$

$$\epsilon = 0.$$

$$\Phi'' + \frac{9-4n}{2n}\Phi' = 0.$$
$$\delta' = 0.$$



# Summary



- ❖ Alternative scenario on cosmology will give **huge** effects on astronomy, cosmology, and particle physics.
- ❖ Some of MG and DE models can satisfy current observations.
- ❖ Geometric tests are not enough to separate MG with DE.
- ❖ Need to distinguish possible models with upcoming observations. (lensing, number counts)
- ❖ **Thank You!**